# Mathematics Subject Matter Requirements Matrix (2009)

| **Domains for Mathematics** | **Coursework, Assignments, Assessments** | **Reviewers Comments** |
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| **Domain 1. Algebra**  **1.1 Algebraic Structures**  a. Know why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings) |  |  |
| **1.1 Algebraic Structures**  b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) |  |  |
| **1.1 Algebraic Structures**  c. Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field |  |  |
| **1.2 Polynomial Equations and Inequalities**  a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) |  |  |
| **1.2 Polynomial Equations and Inequalities**  b. Prove and use the following:   * The Rational Root Theorem for polynomials with integer coefficients * The Factor Theorem * The Conjugate Roots Theorem for polynomial equations with real coefficients * The Quadratic Formula for real and complex quadratic polynomials * The Binomial Theorem |  |  |
| **1.2 Polynomial Equations and Inequalities**  c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra |  |  |
| **1.3 Functions**  a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) |  |  |
| **1.3 Functions**  b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) |  |  |
| **1.3 Functions**  c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) |  |  |
| **1.4 Linear Algebra**  a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products |  |  |
| **1.4 Linear Algebra**  b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) |  |  |
| **1.4 Linear Algebra**  c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) |  |  |
| **Domain 2. Geometry**  **2.1 Parallelism**   1. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) |  |  |
| **2.1 Parallelism**  b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic) |  |  |
| **2.2 Plane Euclidean Geometry**   1. Prove theorems and solve problems involving similarity and congruence |  |  |
| **2.2 Plane Euclidean Geometry**  b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) |  |  |
| **2.2 Plane Euclidean Geometry**  c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) |  |  |
| **2.2 Plane Euclidean Geometry**  d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8) |  |  |
| **2.2 Plane Euclidean Geometry**  e. Use techniques in coordinate geometry to prove geometric theorems |  |  |
| **2.3 Three-Dimensional Geometry**  a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions |  |  |
| **2.3 Three-Dimensional Geometry**  b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres) |  |  |
| **2.4 Transformational Geometry**   1. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection) |  |  |
| **2.4 Transformational Geometry**  b. Understand and prove the basic properties of dilations (e.g., similarity transformations or change of scale) |  |  |
| **Domain 3. Number Theory**  **3.1 Natural Numbers**   1. Prove and use basic properties of natural numbers (e.g., properties of divisibility) |  |  |
| **3.1 Natural Numbers**  b. Use the Principle of Mathematical Induction to prove results in number theory |  |  |
| **3.1 Natural Numbers**  c. Know and apply the Euclidean Algorithm |  |  |
| **3.1 Natural Numbers**  d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational) |  |  |
| **Domain 4. Probability and Statistics**  **4.1 Probability**   * 1. Prove and apply basic principles of permutations and combinations |  |  |
| **4.1 Probability**  b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles) |  |  |
| **4.1 Probability**  c. Use and explain the concept of conditional probability |  |  |
| **4.1 Probability**  d. Interpret the probability of an outcome |  |  |
| **4.1 Probability**  e. Use normal, binomial, and exponential distributions to solve and interpret probability problems |  |  |
| **4.2 Statistics**  a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions |  |  |
| **4.2 Statistics**  b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions  Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results |  |  |
| **4.2 Statistics**  c. Know the method of least squares and apply it to linear regression and correlation |  |  |
| **4.2 Statistics**  d. Know and apply the chi-square test |  |  |
| **Domain 5. Calculus\***  **5.1 Trigonometry**  a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2x + cos2x = 1 and that this identity leads to 1 + tan2x = sec2x and 1 + cot2x = csc2x |  |  |
| **5.1 Trigonometry**  b. Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle) |  |  |
| **5.1 Trigonometry**  c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) |  |  |
| **5.1 Trigonometry**  d. Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) |  |  |
| **5.1 Trigonometry**  e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) |  |  |
| **5.2 Limits and Continuity**  a. Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit |  |  |
| **5.2 Limits and Continuity**  b. Show that a polynomial function is continuous at a point |  |  |
| **5.2 Limits and Continuity**  c. Know and apply the Intermediate Value Theorem, using the geometric implications of continuity |  |  |
| **5.3 Derivatives and Applications**  a. Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative |  |  |
| **5.3 Derivatives and Applications**  b. Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton’s method, and instantaneous rate of change)  Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle’s Theorem, the Mean Value Theorem, and L’Hopital’s rule |  |  |
| **5.3 Derivatives and Applications**  c. Use the derivative to solve rectilinear motion, related rate, and optimization problems |  |  |
| **5.3 Derivatives and Applications**  d. Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity) |  |  |
| **5.3 Derivatives and Applications**  e. Solve separable first-order differential equations and apply them to growth and decay problems |  |  |
| **5.4 Integrals and Applications**  a. Derive definite integrals of standard algebraic functions using the formal definition of integral |  |  |
| **5.4 Integrals and Applications**  b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) |  |  |
| **5.4 Integrals and Applications**  c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives |  |  |
| **5.4 Integrals and Applications**  d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures |  |  |
| **5.5 Sequences and Series**  a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) |  |  |
| **5.5 Sequences and Series**  b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests) |  |  |
| **5.5 Sequences and Series**  c. Calculate Taylor series and Taylor polynomials of basic functions |  |  |
| **Domain 6. History of Mathematics\***  **6.1 Chronological and Topical** **Development of Mathematics**  a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society |  |  |
| **6.1 Chronological and Topical** **Development of Mathematics**  b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender. |  |  |