



**Mathematics Instructional Added Authorization  
and  
Mathematics Instructional Leadership Specialist  
Credential Program Standards**

**Commission on Teacher Credentialing**

**Standards Adopted  
*September 2010*  
*(Updated March 2011)***

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# Table of Contents

|  |                  |
|--|------------------|
| <b><u>Introduction .....</u></b>   | <b><u>1</u></b>  |
| <b><u>Section 1: Standards of Quality and Effectiveness .....</u></b>              | <b><u>2</u></b>  |
| Preconditions and Common Standards.....  | 11               |
| Program Standards  |                  |
| <u>Mathematics Instructional Added Authorization.....</u>                          | <u>13</u>        |
| <u>Mathematics Instructional Leadership Specialist .....</u>                       | <u>17</u>        |
| <b><u>Section 2: Background Related to Program Standards Development .....</u></b> | <b><u>19</u></b> |

## Introduction

The quality of public education depends substantially on the performance of professional educators. Like all other states, California requires educators to hold credentials granted by the state in order to serve in the public schools. Each state, including California, establishes and enforces standards and requirements for earning credentials for public school service. These certification standards and requirements are among the ways in which states exercise their constitutional responsibility for governing public education.

The quality of professional performance depends heavily on the quality of initial preparation. Each state has a legitimate interest in the quality of training programs for professional educators. In each state, completion of a professional preparation program that has been approved by the state's certification agency is a legal requirement for earning each type of credential, including teaching credentials. State legislatures adopt such requirements because they recognize the critical role of professional preparation in subsequent professional performance.

This handbook has been prepared to guide program sponsors in submitting documents for initial program approval as required by the *Accreditation Framework* and implemented by the Committee on Accreditation (COA) and the Commission on Teacher Credentialing (Commission).

This handbook is organized in four sections.

**Section 1** provides information on the Standards of Quality and Effectiveness for the Mathematics Instructional Added Authorization (MIAA) & Mathematics Instructional Leadership (MIL) Specialist preparation programs. The standards are available in this document and on the Commission's Program Standards web page: <http://www.ctc.ca.gov/educator-prep/STDS-prep-program.html>

**Section 2** provides the background for MIAA and MIL program standards development

**Section 3** provides submission/transmittal guidelines for program sponsors preparing documentation for initial program approval

The Commission is grateful to all the members of the profession who participated in the development of these program standards.

## Section 1: Standards of Quality and Effectiveness

California state law authorizes the Commission on Teacher Credentialing to set standards and requirements for preparation of California teachers. The *Accreditation handbook* includes three types of standards:

- **Preconditions** established by State law or Commission policy must be met as a prerequisite to program accreditation. A precondition is a requirement for initial and continued program approval. Unlike standards, preconditions specify requirements for program compliance, not program quality. Commission staff members determine whether a program complies with the adopted preconditions on the basis of a program document provided by the college or university. In the program review sequence, a program that meets all preconditions is eligible for a more intensive review to determine whether the program's quality satisfies the Commission's standards.
- **Common Standards** of program quality and effectiveness apply to all certificate and credential programs. This category includes standards regarding the overall leadership and climate for educator preparation within the unit at an institution, as well as standards pertaining to quality features that are common to all programs such as resources, coordination, admissions and advisement. The Common Standards are available at <http://www.ctc.ca.gov/educator-prep/STDS-common.html>.
- **Program Standards** address the quality of program features that are specific to a credential, such as program design, curriculum, field experiences, and knowledge and skills to be demonstrated by candidates in the specific credential area. When institutions prepare for continuing accreditation reviews, they may consider from among three Commission-approved options for program-specific standards. The three options are: (1) California Program Standards, (2) National or Professional Program Standards, and (3) Experimental Program Standards. Different options may be exercised by different credential programs at an institution.

Standards are statements of program quality that must be fulfilled for initial or continued approval of teacher preparation programs by the Commission. The Commission adopts preconditions and program standards and in September 2010 the Commission adopted these preconditions and program standards. In each standard the Commission has detailed the minimum programmatic inputs and candidate competencies required for approval of a program.

The Commission determines whether a program satisfies a standard on the basis of an intensive review of all available information related to the standard. Program reviewers selected by the Executive Director must find that a program meets each Commission adopted standard. When the program has been deemed to meet all adopted standards, the program is recommended for approval to the COA, and the COA approves the program.

This handbook specifically addresses program standards for programs leading to a MIAA or MIL. A Word version of these program standards is available at: <http://www.ctc.ca.gov/educator-prep/STDS-prep-program.html>.

## **Elaboration of Mathematics Subject Matter Requirements**

The curriculum of the program addresses the Subject Matter Requirements and standards of program quality as set forth in this document. The following is the mathematics background the candidate is expected to know prior to admission to the Mathematics Instructional Added Authorization program.

### **K–Pre-Algebra**

#### **Subject Matter Domain 1: Algebra**

Candidates demonstrate an understanding of the foundations of the algebra contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of algebra in elementary and middle school settings, candidates are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations.

#### **1.1 Linear and Higher Order Polynomial Equations and Linear Inequalities**

- a. Understand graphs of linear equations and inequalities
- b. Prove and use the Quadratic Formula
- c. Understand and use The Division Algorithm and The Factor Theorem
- d. Analyze and simplify polynomial expressions and solve polynomial equations with real coefficients

(Mathematics Content Standards for California Public Schools, Grade 4, Algebra and Functions: 1.0; Measurement and Geometry: 2.0; Grade 5, Algebra and Functions: 1.0; Grade 6, Algebra and Functions: 1.0, 2.0; Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Common Core State Standards for Mathematics, 6.EE, 7.RP, 7.EE, 8.EE, 8.F, A-SSE, A-APR, A-CED, A-REI)

#### **1.2 Functions**

- a. Analyze and derive general properties of functions (i.e., domain and range and differences between relations and functions)
- b. Analyze properties of polynomial and absolute value functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 3, Algebra and Functions: 2.0; Grade 4, Algebra and Functions: 1.0; Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Common Core State Standards for Mathematics, 6.EE, 7.EE, 7.NS, 8.F, A-SSE, A-APR, A-CED, A-REI, F-IF, F-BF, F-LE)

## **Subject Matter Domain 2: Geometry**

Candidates demonstrate an understanding of the foundations of the geometry contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of geometry in elementary and middle school settings, candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry.

### **2.1 Parallelism**

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)

(Mathematics Content Standards for California Public Schools, Grade 3, Measurement and Geometry: 2.0; Grade 4, Measurement and Geometry: 3.0; Grade 5, Measurement and Geometry: 2.0; Grade 6, Measurement and Geometry: 2.0; Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0; Common Core State Standards for Mathematics, 4.G, 5.G, G-CO)

### **2.2 Plane Euclidean Geometry**

- a. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, trigonometric ratios, Triangle Inequality, the Pythagorean Theorem and its converse)
- b. Understand, apply, and justify properties of polygons and circles (e.g., analyze figures in terms of area and perimeter; derive the area formulas for regular polygons from the area of a triangle; solve problems using the circumference and area formulas of a circle)
- c. Use techniques in coordinate geometry (e.g., distance formula, midpoint formula) to solve geometric problems
- d. Prove simple theorems and solve problems involving similarity and congruence (e.g., base angles of isosceles triangles are congruent)

(Mathematics Content Standards for California Public Schools, Kindergarten, Measurement and Geometry: 1.0, 2.0; Grade 1, Measurement and Geometry: 1.0, 2.0; Grade 2, Measurement and Geometry: 1.0, 2.0; Grade 3, Measurement and Geometry: 1.0, 2.0, 3.0; Grade 4, Algebra and Functions: 1.0; Measurement and Geometry: 1.0-3.0; Grade 5 Measurement and Geometry: 1.0, 2.0; Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Common Core State Standards for Mathematics, K.MD, K.G, 1.MD, 1.G, 2.MD, 2.G, 3.MD, 3.G, 4.MD, 4.G, 5.MD, 5.G, 6.G, 7.G, 8.G, G-CO, G-SRT, G-GPE, G-GMD, GC, G-MG)

### **2.3 Three-Dimensional Geometry**

- a. Understand how two or more objects are related in space (e.g., skew lines, the possible ways three planes might intersect)
- b. Understand relationships among three-dimensional objects and apply formulas to find the volume and surface area of prisms, pyramids, cylinders, cones, and spheres

(Mathematics Content Standards for California Public Schools, Grade 2, Measurement and Geometry: 2.0; Grade 3, Measurement and Geometry: 2.0; Grade 4, Measurement and Geometry, 1.0, 3.0; Grade 5, Measurement and Geometry, 1.0, 2.0; Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Common Core State Standards for Mathematics, 5.MD, 6.G, 7.G, 8.G, GMD, G-MG)

### **Subject Matter Domain 3: Number and Quantity**

Candidates demonstrate an understanding of the real number system and its subsets, as used in elementary and middle school settings, and contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*.

#### **3.1 Real Number System and Its Subsets**

- a. Use basic properties of natural numbers (e.g., properties of divisibility) and understand prime and composite numbers
- b. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple; show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime; prove that the square root of any number, not a perfect square number, is irrational)
- c. Understand the relative magnitude of real numbers and how to order real numbers presented in a variety of ways
- d. Apply basic properties of rational numbers and use fractions, decimals, and percents to solve problems
- e. Analyze proportional relationships and use them to solve real-world and mathematical problems
- f. Understand basic properties of real numbers (e.g., distributive, commutative, associative) and work with radicals and exponents

(Mathematics Content Standards for California Public Schools, Kindergarten, Number Sense: 1.0-3.0; Grade 1, Number Sense:1.0-3.0; Grade 2, Number Sense: 1.0-6.0; Grade 3, Number Sense: 1.0-3.0; Grade 4, Number Sense: 1.0-4.0; Grade 5, Number Sense: 1.0, 2.0; Grade 6, Number Sense: 1.0, 2.0; Grade 7, Number Sense: 1.0, 2.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry:1.0; Common Core State Standards for Mathematics; K.CC, K.OA, K.NBT, K.MD, 1.OA, 1.NBT, 2.OA, 2.NBT, 2.MD, 3.OA, 3.NBT, 3.NF, 3.MD, 4.OA, 4.NBT, 4.NF, 4.MD, 5.OA, 5.NBT, 5.NF, 5.ND, 6.RP, 6.NS, 6.EE, 7.RP, 7.NS, 7.EE, 8.NS, 8.EE, N-RN, N-Q)

### **Subject Matter Domain 4: Probability and Statistics**

Candidates demonstrate an understanding of the statistics and probability distributions contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of probability and statistics in elementary and middle school settings, candidates solve problems and make inferences using statistics and probability distributions.

#### **4.1 Probability**

- a. Understand and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability

(Mathematics Content Standards for California Public Schools, Grade 3, Statistics, Data Analysis, and Probability: 1.0; Grade 4, Statistics, Data Analysis, and Probability: 2.0; Grade 5, Statistics, Data Analysis, and Probability: 1.0; Grade 6, Statistics, Data Analysis, and Probability: 3.0; Probability and Statistics: 1.0-4.0; Common Core State Standards for Mathematics, 7.SP, S-CP, S-MD)

## 4.2 Statistics

- Understand, determine, and interpret the mean, median, and mode of discrete distributions
- Understand, determine, and interpret the range of both discrete and continuous distributions
- Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results

(Mathematics Content Standards for California Public Schools, Kindergarten, Statistics, Data Analysis, and Probability: 1.0; Grade 1, Statistics, Data Analysis, and Probability: 1.0; Grade 2, Statistics, Data Analysis, and Probability: 1.0; Grade 3, Statistics, Data Analysis, and Probability: 1.0; Grade 4, Statistics, Data Analysis, and Probability: 1.0; Grade 5, Statistics, Data Analysis, and Probability: 1.0; Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Common Core State Standards for Mathematics, 2.MD, 3.MD, 4.MD, 5.MD, 6.SP, 7.SP, 8.SP, S-ID, S-IC)

## K–Algebra 1

### Subject Matter Domain 1: Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)* and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of algebra in elementary and middle school settings, candidates are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations.

### 1.1 Polynomial Equations and Inequalities

- Understand graphs of linear equations
- Know why graphs of linear inequalities are half planes and apply this fact (e.g., linear programming)
- Prove and use the Quadratic Formula for real and complex quadratic polynomials
- Understand and use the following: The Division Algorithm, The Remainder Theorem, The Factor Theorem, and The Conjugate Roots Theorem (the last of these theorems limited to quadratic equations with real coefficients)
- Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 4, Algebra and Functions: 1.0; Measurement and Geometry: 2.0; Grade 5, Algebra and Functions: 1.0; Grade 6, Algebra and Functions: 1.0, 2.0; Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-10.0; Common Core State Standards for Mathematics, 6.EE, 7.RP, 7.EE, 8.EE, 8.F,A-SSE, A-APR, A-CED, A-REI, N-CN)

### 1.2 Functions

- Analyze and derive general properties of functions (i.e., domain and range, one-to-one, inverses, composition, and differences between relations and functions)
- Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 3, Algebra and Functions: 2.0; Grade 4, Algebra and Functions: 1.0; Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-10.0, 24.0, 25.0; Common Core State Standards for Mathematics, 6.EE, 7.EE, 7.NS, 8.EE, 8.F, A-SSE, A-APR, A-CED, A-REI F-IF, F-BF, F-LE)

## **Subject Matter Domain 2: Geometry**

Candidates demonstrate an understanding of the foundations of the geometry contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of geometry in elementary and middle school settings, candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry.

### **2.1 Parallelism**

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)

(Mathematics Content Standards for California Public Schools, Grade 3, Measurement and Geometry: 2.0; Grade 4, Measurement and Geometry: 3.0; Grade 5, Measurement and Geometry: 2.0; Grade 6, Measurement and Geometry: 2.0; Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0; Common Core State Standards for Mathematics, 4.G, 5.G, G-CO)

### **2.2 Plane Euclidean Geometry**

- a. Prove simple theorems and solve problems involving similarity and congruence (e.g., base angles of isosceles triangles are congruent)
- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, Triangle Inequality, the Pythagorean Theorem and its converse)
- c. Understand, apply, and justify properties of polygons and circles (e.g., analyze figures in terms of area and perimeter, derive the area formulas for regular polygons from the area of a triangle, solve problems using the circumference and area formulas of a circle)
- d. Use techniques in coordinate geometry (e.g., distance formula, midpoint formula, slope criteria for parallel and perpendicular lines) to solve geometric problems and prove theorems

(Mathematics Content Standards for California Public Schools, Kindergarten, Measurement and Geometry: 1.0, 2.0; Grade 1, Measurement and Geometry: 1.0, 2.0; Grade 2, Measurement and Geometry: 1.0, 2.0; Grade 3, Measurement and Geometry: 1.0, 2.0, 3.0; Grade 4, Algebra and Functions: 1.0; Measurement and Geometry: 1.0-3.0; Grade 5 Measurement and Geometry: 1.0, 2.0; Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Common Core State Standards for Mathematics, K.MD, K.G, 1.MD, 1.G, 2.MD, 2.G, 3.MD, 3.G, 4.MD, 4.G, 5.MD, 5.G, 6.G, 7.G, 8.G, G-CO, G-SRT, G-GPE, G-GMD, GC, G-MG)

### **2.3 Three-Dimensional Geometry**

- a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects (e.g., derive the volume and surface area formulas for prisms, pyramids, cylinders)

(Mathematics Content Standards for California Public Schools, Grade 2, Measurement and Geometry: 2.0; Grade 3, Measurement and Geometry: 2.0; Grade 4, Measurement and Geometry, 1.0, 3.0; Grade 5, Measurement and Geometry, 1.0, 2.0; Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Common Core State Standards for Mathematics, 5.MD, 6.G 7.G, 8.G, GMD, G-MG)

### **Subject Matter Domain 3: Number and Quantity**

Candidates demonstrate an understanding of the complex number system and its subsets, as used in school settings, and contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*.

#### **3.1 Complex Number System and its Subsets**

- a. Use basic properties of natural numbers (e.g., properties of divisibility)
- b. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)
- c. Understand and apply the properties of the rational and real numbers (e.g., the closure, commutative, associative, identity, inverse, and distributive properties; properties of equality, properties of order)
- e. Know that the rational numbers and real numbers can be ordered and that complex numbers can not

(Mathematics Content Standards for California Public Schools, Kindergarten, Number Sense: 1.0-3.0; Grade 1, Number Sense: 1.0-3.0; Grade 2, Number Sense: 1.0-6.0; Grade 3, Number Sense: 1.0-3.0; Grade 4, Number Sense: 1.0-4.0; Grade 5, Number Sense: 1.0, 2.0; Grade 6, Number Sense: 1.0, 2.0; Grade 7, Number Sense: 1.0, 2.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 5.0, 6.0; Common Core State Standards for Mathematics, K.CC, K.OA, K.NBT, K.MD, 1.OA, 1.NBT, 2.OA, 2.NBT, 2.MD, 3.OA, 3.NBT, 3.NF, 3.MD, 4.OA, 4.NBT, 4.NF, 4.MD, 5.OA, 5.NBT, 5.NF, 5.ND, 6.RP, 6.NS, 6.EE, 7.RP, 7.NS, 7.EE, 8.NS, 8.EE, N-RN, N-Q, N-CN)

### **Subject Matter Domain 4: Probability and Statistics**

Candidates demonstrate an understanding of the statistics and probability distributions contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*, and the *Common Core State Standards for Mathematics (2010)*. To ensure a clear understanding of the conceptual underpinnings of probability and statistics in elementary and middle school settings, candidates solve problems and make inferences using statistics and probability distributions.

#### **4.1 Probability**

- a. Understand and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability

(Mathematics Content Standards for California Public Schools, Grade 3, Statistics, Data Analysis, and

Probability: 1.0; Grade 4, Statistics, Data Analysis, and Probability: 2.0; Grade 5, Statistics, Data Analysis, and Probability: 1.0; Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0; Common Core State Standards for Mathematics, 7.SP, S-CP, S-MD)

#### **4.2 Statistics**

- a. Understand, determine, and interpret the mean, median, and mode of discrete distributions
- b. Understand, determine, and interpret the range of both discrete and continuous distributions
- c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results

(Mathematics Content Standards for California Public Schools, Kindergarten, Statistics, Data Analysis, and Probability: 1.0; Grade 1, Statistics, Data Analysis, and Probability: 1.0; Grade 2, Statistics, Data Analysis, and Probability: 1.0; Grade 3, Statistics, Data Analysis, and Probability: 1.0; Grade 4, Statistics, Data Analysis, and Probability: 1.0; Grade 5, Statistics, Data Analysis, and Probability: 1.0; Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Common Core State Standards for Mathematics, 2.MD, 3.MD, 4.MD, 5.MD, 6.SP, 7.SP, 8.SP, S-ID, S-IC)

## Preconditions

Preconditions are requirements that must be met in order for an accrediting association or licensing agency to consider accrediting a program sponsor or approving its programs or schools. Some preconditions are based on state laws, while other preconditions are established by Commission policy. Institutions are required to submit information related to the Preconditions to the Commission at three points in the accreditation system: 1) during year one of the accreditation cycle, 2) during year four of the accreditation cycle and 2) upon submitting a new program proposal.

There are essentially two kinds of preconditions. The first are the Commission's ten General Institutional Preconditions. These apply to all professional preparation programs—teacher and services credential preparation programs. These preconditions do not apply to subject matter programs.

The second type of preconditions apply to particular kinds of credential preparation programs. There are four Program Specific Preconditions that apply to all types of educator preparation programs. In addition, there are preconditions for many types of educator preparation programs. All program sponsors must respond to each of the applicable preconditions.

Click to the following link to locate the preconditions. <http://www.ctc.ca.gov/educator-prep/standards/Standards-Preconditions.pdf>.

*The required Preconditions for the **Mathematics Instructional Added Authorization Program** are General Preconditions 1-10, and Program Specific Preconditions 1-4, and 5. The required preconditions for **Mathematics Instructional Leadership Specialist Program** are: General Preconditions 1-10; Program Specific Preconditions 1-4 and 5-7.*

## Common Standards

The Common Standards address issues of institutional infrastructure, stability, and processes that are designed to ensure that the implementation of all approved programs is successful and meets all standards. Consequently, there is a single response to the nine Common Standards that reflects the institution's support of each of its educator preparation programs. Institutions are required to submit information related to the Common Standards to the Commission at two points in the accreditation system: 1) during year 5 of the accreditation cycle- the year before the accreditation site visit; and 2) upon submitting a new program proposal.

The institution must develop **one response** to the Common Standards that reflects institutional support for all approved educator preparation programs. In other words, individual programs do

not respond to the Common Standards. The Common Standards document is inclusive of the entire unit, consequently only one Common Standards document will be submitted to the CTC for each approved institution/program sponsor regardless of how many approved programs are offered. Click on the following link to locate the Common Standards

<http://www.ctc.ca.gov/educator-prep/STDS-common.html>.

If the institution's Common Standards are up to date and the institution submits a new program proposal, the institution must complete an addendum to the Common Standards that assures the Commission that the institution will support the proposed program in the same way it has supported other educator preparation programs. Click on the following link to locate the Common Standards Addendum <http://www.ctc.ca.gov/educator-prep/program-standards.html>.

The Common Standards Glossary should be consulted for definitions of any of the terms found in *italics* in the Common Standards.

# Mathematics Instructional Added Authorization Program Standards

## Category A: Program Design

### Standard 1: Program Design

The preparation program and any prerequisites include a purposeful, interrelated, and developmentally-designed sequence of coursework and field experiences. The design of the program follows an explicit statement of program philosophy and purpose based on a sound rationale informed by theory, research, and practice. It effectively coordinates and articulates expertise in integrating and applying K-Pre-Algebra and/or K-Algebra I content knowledge, specialized mathematical knowledge for teaching and thinking, and pedagogical knowledge and practices for teaching mathematics.

The sponsoring institution demonstrates a commitment to candidate preparation by providing appropriate support for the program. The program has a qualified leadership team with expertise in mathematics content, mathematics education, teacher education, and teacher leadership.

The program provides extensive opportunities for candidates to demonstrate mathematical and pedagogical content knowledge and skills to support effective mathematics instruction and student learning. Coursework and fieldwork address the complex interplay of math content and pedagogy in effective teaching. Candidates are prepared to enhance mathematical development for all students<sup>0</sup> including English learners, students with disabilities, students who are gifted and talented, and students at risk. Candidates are prepared to collaborate and co-teach with other math teachers as well as teachers of other subject matter disciplines. The program includes a planned process of comprehensive assessments ensuring that candidates are prepared to teach K through Pre-Algebra or K through Algebra I. In addition, its design ensures that candidates are equipped to understand the challenges of developing mathematics literacy among California's diverse student and teaching population.

## Category B: Curriculum and Fieldwork

These three mutually supportive domains are defined by the following seven elements and organized into two standards, which provide structure for the program design:

|                                       |   |   |
|---------------------------------------|---|---|
| <b>Mathematical Content Knowledge</b> | <b>Specialized Mathematical Knowledge for Teaching and Thinking</b> | <b>Pedagogical Knowledge and Practices for Teaching Mathematics</b> |
|---------------------------------------|---|---|

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<sup>0</sup> All "students" refers to, but is not limited to, struggling students, English learners, gifted and talented students, and students with special needs. Program Sponsors will need to address all levels whenever the word "students" is used in this document.

| <b>Mathematical Content Knowledge</b>  | <b>Specialized Mathematical Knowledge for Teaching and Thinking</b>  | <b>Pedagogical Knowledge and Practices for Teaching Mathematics</b>   |
|--|--|---|
| Admission to the program is contingent on candidate mastery of mathematics as determined by the program based on the math content outlined in the preconditions. | <ol style="list-style-type: none"> <li>1. Students' mathematical thinking</li> <li>2. Mathematical representation</li> <li>3. Mathematical language</li> </ol> | <ol style="list-style-type: none"> <li>4. Mathematics curriculum</li> <li>5. Instructional planning</li> <li>6. Classroom discourse</li> <li>7. Assessment</li> </ol> |

### **Standard 2: Specialized Mathematics Knowledge for Teaching and Thinking**

The program provides opportunities for candidates to develop Mathematical Knowledge for Teaching and understand mathematics content, consistent with the candidate's level of certification. The program provides opportunities for candidates to develop advanced competency to:

1. Plan and implement instruction that includes differentiation, accommodations, and interventions and is based on students' mathematical thinking by:
  - a. Using error analysis processes to review and understand students' misconceptions and to distinguish whether a specific error reflects a misunderstanding of the mathematical process or a need for language development in the area of mathematical concepts
  - b. Understanding the order of presentation of mathematical concepts that lead to student proficiency in mathematics
  - c. Analyzing the learning trajectories of individual students
  - d. Explaining standard and alternative algorithms and solution strategies
2. Use a variety of appropriate methods of mathematical representation, including technology, oral language, written symbols, pictures, concrete materials/models, real-world situations, while also:
  - a. Identifying and understanding strengths and limitations of various representations of mathematical concepts
  - b. Linking representations to underlying mathematical theories and to other representations.
3. Use and understand the correct usage of mathematical language by:
  - a. Analyzing mathematical vocabulary in the context of mathematical concepts when listening and responding to students' questions and comments
  - b. Evaluating their own and colleagues' presentation of mathematical definitions and terms, as well as considering students' culture, language, and cognitive needs when using academic language to scaffold instruction

### **Standard 3: Pedagogical Knowledge and Practices for Teaching Mathematics**

The program provides candidates with opportunities to develop advanced practices in the use of students' mathematical thinking, mathematical representation, mathematical language, mathematics curriculum, instructional planning, classroom discourse, and assessment. The program will provide opportunities for candidates to develop advanced skills in evaluating, planning, and implementing appropriate interventions to increase student achievement.

The program will provide opportunities for candidates to develop the type of pedagogical expertise needed to modify curriculum to address the specific needs of diverse groups of students, including but not limited to struggling students, English language learners, gifted and talented students, and students with special needs.

Candidates must be able to demonstrate advanced competency to:

1. Plan and implement mathematics instruction in developmentally and culturally responsive ways to meet specific student needs, including the ability to
  - a. Analyze and adapt resources, technologies, and standards-aligned instructional materials, including adopted materials, for targeted audiences based on students' cultural, linguistic, and cognitive development
  - b. Identify and connect concepts that are fundamental to learning mathematics, such as place value, fractions, real numbers and algebra
2. Plan instruction that supports students' learning of mathematics by:
  - a. Selecting and developing student learning tasks that enable teachers to understand the conjectures and generalizations that students make
  - b. Aligning instructional goals, assessments, instructional strategies, and practice (e.g., assignments, homework) using SDAIE strategies as appropriate
  - c. Designing and implementing flexible grouping strategies (homogeneous, semi-homogeneous, heterogeneous, large group, small group, and individual learning) according to students' needs and level of achievement
  - d. Focusing on the mathematics content standards and the key concepts within the standards
  - e. Collaborating with individual teachers (pre-service, novice, and experienced) through co-planning and co-teaching to improve student learning
3. Develop strategies for classroom discourse by being able to:
  - a. Facilitate student to student interaction
  - b. Analyze questioning strategies to lead discussions that actively involve all students
  - c. Select culturally appropriate examples and reframe problems to encourage students' deep understanding within a mathematical context
  - d. Advance and cultivate positive attitudes toward mathematics; encouraging curiosity, flexibility, and persistence in solving mathematical problems
4. Use assessments for:
  - a. Identifying gaps in students' knowledge and for designing instruction to bridge the gaps, language assessments for identifying gaps in understanding mathematics terms, summative assessments and standardized assessments for measuring student growth
  - b. Guiding instruction and developing curriculum that is targeted, accommodated, and differentiated for intensive intervention as necessary
  - c. Communicating progress to students, parents, colleagues, and other appropriate service providers
  - d. Deriving demographic, process, and outcome data at the student, school, and district levels to support informed decisions in designing targeted instruction that promotes students equitable access to learn high-level mathematics

**Standard 4: Field Experiences**

Programs facilitate individualized and balanced field experiences that provide candidates with timely and ongoing feedback to guide improvements in practice as described in Category B. These field experiences are integrated into coursework and are aligned with the candidate competence standard.

The guided field experiences extend candidates' understandings of the three domains and their elements. The candidate is provided substantive opportunities to observe and practice each of the proficiencies described in Category B. The fieldwork component will include the following grade spans: Kindergarten through Grade 3 and Grade 4 through Grade 7 for the candidates that are prepared to teach K through Pre-Algebra. In addition, candidates prepared to teach K through Algebra I also have field experience in Algebra 1. The program collaborates with local educational agencies in providing guidance, site-based support, and coordination of field experiences to ensure the candidate has successful experiences working with English learners, students with disabilities, students who are gifted and talented, and students at risk.

**Category C: Candidate Competence****Standard 5: Determination of Candidate Competence**

Program sponsors use multiple measures to determine that each candidate has demonstrated competence across the proficiencies described in Category B, including advanced level culminating projects to demonstrate professional competency. Program options for advanced level culminating projects to demonstrate professional competency may include, but are not limited to, professional presentations, action research, designing curriculum, and school, district, or county collaborative projects.

# Mathematics Instructional Leadership (MIL) Specialist Proposed Program Standards

## Category A: Program Design

### Standard 1: Program Design

The preparation programs and their prerequisites include a purposeful, interrelated, developmentally designed sequence of coursework and field experiences. Programs provide integrated coursework and fieldwork through a model that enables each candidate to demonstrate proficiency as a MIL Specialist. The program includes a planned process for the comprehensive assessment of candidates in the following areas:

1. Understand practitioner research and encourage teachers to use it in their practice
2. Design and implement professional development that engages teachers, administrators, and parents while promoting student engagement and achievement in mathematics
3. Analyze and use data to design solutions to the challenges of developing mathematical literacy among California's diverse population
4. Lead a professional community of practitioners to promote student engagement and achievement in mathematics and minimize the achievement gap

## Category B: Curriculum and Fieldwork

### Standard 2: Leadership Knowledge and Skills for the Mathematics Instructional Leader:

The candidate will facilitate the use of a variety of appropriate content-based learning materials and learning strategies that recognize students as active learners, understand the importance of reflection and inquiry, emphasize the quality of student application and performance, utilize appropriate and effective technology, and accelerate mathematics achievement for all students, including English Learners, students with special needs, gifted and talented students, and students at risk. Candidates will guide and support the long-term professional development of staff, consistent with the ongoing effort to improve the learning of all students, relative to the content standards, and provide opportunities for all members of the school community to develop and use skills in distributed leadership and shared responsibility. The candidates will utilize multiple assessments, including assessments that are sensitive to the learning needs of special populations in terms of language, culture, and language, processing and cognitive difficulties, to evaluate student learning in an ongoing process focused on improving the academic performance of each student.

Specifically, the program prepares candidates to demonstrate expertise in the following four areas: research-supported mathematics teaching, learning and coaching, professional

development and learning, using data to inform student instruction and professional development, and developing professional learning communities.

**Standard 3: Fieldwork Integrated with Coursework for Mathematics Instructional Leadership:**

Candidates are provided extensive opportunities to observe, acquire, and use appropriate pedagogical content knowledge for teaching, coaching, and mentoring, and to acquire skills to design and implement innovative processes that are research supported, including uses of technology. Programs provide candidates with timely and on-going feedback to guide improvement in practice through action research connected to instruction, program design, assessment, and leadership. These field experiences are embedded in coursework and aligned with the program assessment standards. The program provides opportunities for candidates to collaborate with local educational agencies in providing guidance, site-based support, and supervision of field experiences.

Programs facilitate individualized and balanced field experiences that provide candidates with timely and ongoing feedback to guide improvements in practice as described in Category B. These field experiences are integrated into coursework and are aligned with the candidate competence standard. Candidates will support opportunities for all members of the school community to develop and use skills in collaboration, distributed leadership, and shared responsibility in ways that are sensitive to students' families' cultures.

MIL Specialist credential candidates must also demonstrate the capacity to analyze the effectiveness of their own practices in terms of the direct impact of their practices on the people with whom they work (e.g., students, teachers, parents, administrators, and community members), in part, by minimizing the achievement gap, and the real or potential impact of their practices on research of students and student learning of mathematics.

**Category C: Assessment of Candidate Competence**

**Standard 4: Determination of Candidate Professional Competence for the Mathematics Instructional Leader:**

Program sponsors may provide any combination of advanced level culminating projects through which candidates demonstrate professional expertise and competency that reflect the candidates' capacity to tailor assessment, instruction, and professional support to the needs of all students, including the special needs of students of different cultures, language levels, and with processing and cognitive difficulties. Candidates will collect field-based evidence throughout the program to demonstrate competence in the four areas of leadership practice at various grade spans (Kindergarten-3, 4-7, Algebra I, Geometry, Algebra II, and Advanced Mathematical study). The evidence should be integrated and demonstrate a professional level of proficiency.

## Section 2: Background Related to Program Standards Development

### The Teaching Mathematics Advisory Panel

The California Teaching Mathematics Advisory Panel (TMAP) was established in 2009 by the Commission and charged with reviewing California's structure for Mathematics credentials in order to support and expand the teaching knowledge and subject matter expertise of California teachers of Mathematics K-12. The panel came to agreement that although teachers of Mathematics have at least a basic level of knowledge and expertise regarding teaching mathematics, there was a pressing need to provide additional support to teachers by increasing access to mathematics teaching expertise on a consistent basis, especially at the elementary and middle grades levels.

### Introduction

Recruiting, training, and retaining effective teachers of mathematics are important and difficult challenges. California public schools have existing and persistent shortages of fully prepared teachers of mathematics throughout all grade levels. California faces three specific challenges in K-12 mathematics: (1) the need for specialized math leadership in the California education system; (2) an increasing need for effective Algebra 1 teachers with a strong foundation in mathematics and pedagogy; and (3) the structure of the multiple subjects credential does not allow K-8 teachers to teach only mathematics.

Many have made the case that practicing elementary school teachers are not adequately prepared to meet the demands for increasing student achievement in mathematics (National Council of Teachers of Mathematics, 2000; National Mathematics Advisory Panel, 2008; National Research Council, 1989). In particular, most elementary teachers are generalists—that is, they study and teach all core subjects, rarely developing in-depth knowledge and expertise with regard to teaching elementary mathematics. Wu (2009) describes the situation in this way:

The fact that many elementary teachers lack the knowledge to teach mathematics with coherence, precision, and reasoning is a systemic problem with grave consequences. Let us note that this is not the fault of our elementary teachers. Indeed, it is altogether unrealistic to expect our generalist elementary teachers to possess this kind of mathematical knowledge (p. 14).

Further, Wu notes a problem of scale in addressing the situation and suggests utilizing a smaller cadre of well-prepared teachers to focus on mathematics at the elementary grades:

Given that there are over 2 million elementary teachers, the problem of raising the mathematical proficiency of all elementary teachers is so enormous as to be beyond comprehension. A viable alternative is to produce a much smaller corps of mathematics teachers with strong content knowledge who would be solely in charge of teaching mathematics at least beginning with grade 4 (p. 14).

This echoes a statement made 20 years earlier in the National Research Council's *Everybody Counts*:

The United States ... continues to pretend—despite substantial evidence to the contrary—that elementary school teachers are able to teach all subjects equally well. It is time that we identify a cadre of teachers with special interests in mathematics and science who would be well prepared to teach young children both mathematics and science in an integrated, discovery-based environment (p. 64).

Over the past two decades, others have made similar recommendations (Battista, 1999; Conference Board of the Mathematical Sciences, 2001, p. 11; Learning First Alliance, 1998; National Council of Teachers of Mathematics, 2000, pp. 375–376; Reys & Fennell, 2003). Recently, the National Mathematics Advisory Panel (2008) noted that “the use of teachers who have specialized in elementary mathematics teaching could be a practical alternative to increasing all elementary teachers’ content knowledge (a problem of huge scale) by focusing the need for expertise on fewer teachers” (p. 44).

### **Evidence of Impact of the Mathematics Instructional Leadership Specialist**

In summarizing their study of reform in schools and districts, Ferrini-Mundy and Johnson (1997) report that the school-based leadership provided by mathematics specialists appeared to be critical to reform. Mathematics specialists “helped (sic) spread ideas, facilitate communications among teachers, plan and initiate staff development, and address political problems with administrators and community members” (p. 119).

Recent studies of states with a corps of math specialists show evidence of a positive impact on student learning. For example, the Vermont Mathematics Initiative (VMI) has built a corps of K–8 mathematics teacher leaders across the state that can support other teachers in their schools and districts (Kessel, 2009, pp. 36–38). Evaluation studies show evidence that VMI has had a major impact on the teachers themselves, their classroom practice, and student achievement. Students in VMI schools outperformed those in control schools, and the achievement gap has narrowed between free- or reduced-lunch eligible students in VMI schools and their non-eligible peers in matched schools (Meyers & Harris, 2008).

### **Issues Identified in the Preparation of Individuals to Teach Mathematics**

Based on the study and discussion of research articles, national panel recommendations, Commission agenda reports, and the California mathematics curriculum framework, the TMAP decided to focus its work in two areas; 1) restructuring and updating the authorizations and standards for the Mathematics Specialist Credential, and 2) expanding the mathematical pedagogy preparation for Multiple Subject credential candidates. These foci were chosen because they provided a mechanism for responding relatively quickly to the critical need for mathematics teaching expertise at the K-8 grade levels (math specialist) and to the long-term needs of K-8 students for mathematically-competent multiple subject teachers.

### **Overview of Revised Mathematics Specialist Credentials**

The TMAP came to agreement that although teachers of mathematics have at least a basic level of knowledge and expertise regarding teaching mathematics, there is a pressing need to provide additional support to teachers by increasing access to mathematics teaching expertise on a consistent basis, especially at the elementary and middle grades levels. The panel felt that access to expertise in mathematics would best be accomplished by updating the former mathematics

specialist credential program standards so that more mathematics specialists could be available statewide to support classroom teachers. As the panel refined its thinking about the needed changes and updates to the mathematics specialist credential, it determined that a structure that parallels the current, nested structure of the reading added authorization and reading instructional leadership specialist credential would be appropriate in the area of mathematics.

First, the panel proposes revising and renaming the existing authorization as the Mathematics Instructional Added Authorization (MIAA). In considering what knowledge, skills, and experience a MIAA holder should have, the panel agreed that an individual with this authorization should be required to complete advanced preparation and fieldwork in both mathematics content and the pedagogy of mathematics above and beyond what is required for the multiple subject teaching credential. In addition, the panel recognized that within the MIAA option, some teachers would have the prerequisite math content mastery to obtain an authorization that would go through but not beyond the level of mathematics typically taught in Kindergarten through Pre-Algebra, whereas other teachers might possess the math content that would authorize the teaching of mathematics Kindergarten through Algebra I.

Further, in addition to the MIC, the panel felt that there should be an authorization to recognize a higher level of specialized skills that would allow an individual to not only provide support to teachers, but also provide leadership at the K-12 level with respect to the teaching and learning of mathematics. This proposed credential is the Mathematics Instructional Leadership (MIL) Specialist Credential.

Following is a further description of the MIAA and the MIL Specialist credential. These program standards are available at: <http://www.ctc.ca.gov/educator-prep/standards/mathematics-specialist.pdf>

### **Mathematics Instructional Added Authorization (MIAA): K-Pre-Algebra and K-Algebra**

Candidates with the prerequisite teaching credential may pursue either of two authorizations through a MIAA program, depending on the mathematics content knowledge of the candidate and the authorization desired:

- (1) The MIAA (K-Pre-Algebra) would apply to candidates who can document mastery of mathematics knowledge, as organized in the California mathematics content standards, from Kindergarten through Algebra I.
- (2) The MIAA (K-Algebra I) would apply to candidates whose mathematics knowledge includes mastery of the California mathematics content standards from Kindergarten through Algebra I, II, and Geometry.

It should be pointed out that the actual knowledge base of the teacher is required to be at a higher level than what the teacher would be authorized to teach.

## Structure of the Mathematics Instructional Added Authorization

| MIC Route     | Precursor Credential | Mathematic Content Knowledge of the Teacher | Authorizes Teaching       |
|---------------|----------------------|---|---------------------------|
| K-Pre-Algebra | Teaching Credential  | K-7 through Algebra I                       | Kindergarten -Pre-Algebra |
| K-Algebra I   |                      | K-7, Algebra I, Geometry, and Algebra II    | Kindergarten-Algebra I    |

The MIAA would be an authorization beyond the preliminary, life, or clear multiple subject teaching credentials. It is anticipated that the holder of either MIAA authorization would play a major role in bridging the existing achievement gap as he or she would have expertise in curriculum design, coaching teachers, designing and implementing intensive interventions, and teaching teachers to effectively intervene, accommodate, and differentiate their mathematics instruction to increase student engagement and proficiency in mathematics from Kindergarten through Pre-Algebra or Algebra I.

The holder of either MIAA authorization would need to have the knowledge and skills needed to provide leadership in a comprehensive Kindergarten through Pre-Algebra/Algebra I mathematics program that addresses the instructional needs of English Learners, students with disabilities, gifted and talented students, and students at risk. Additionally, the holder of either MIAA authorization could potentially teach mathematics from Kindergarten through Pre-Algebra/Algebra I in a departmentalized setting. The impact of the holder of either MIAA authorization might include, but not be limited to, increasing:

- student proficiency in K-Pre-Algebra/Algebra I and closing the achievement gap by providing math instructional leadership to schools, districts, and counties in areas such as curriculum design, coaching, intensive interventions, accommodation, and differentiation
- expertise in teaching K-Pre-Algebra/Algebra I subject matter in either a departmentalized or self-contained setting to all students, including English Learners, students with disabilities, gifted and talented students and students at risk
- the number of highly qualified K-Pre-Algebra/Algebra I teachers in departmentalized settings.

### **Mathematics Instructional Leadership (MIL) Specialist Credential**

Individuals must complete the Mathematics Instructional Added Authorization program before they would be eligible for the MIL Specialist credential as this credential is built upon the MIC. The proposed new MIL Specialist credential would provide experienced teachers the skills required to:

- promote more effective teaching and learning of mathematics PreK-12
- provide the leadership and a vision for mathematics instruction for schools, districts, and county offices
- fulfill a need in the field for a cadre of mathematics teacher leaders who can connect content level and coaching expertise with school, district, and/or county leadership

Programs preparing MIL Specialist credential candidates would include advanced preparation and fieldwork in:

- effectively connecting action research and mentoring/coaching skills with theoretical research to bridge the theory and practice divide in mathematics teaching and learning
- designing and implementing a school and/or district professional development system that involves teachers and administrators in working collaboratively to increase student engagement and learning in mathematics
- analyzing and using student, school, district, county, state, and college/university data to inform school and district program design to increase the number of students who are college-ready and to reverse the pervasive achievement gap
- leading a professional community of practice

### **Guidance for Prospective Program Sponsors/ Preconditions Established by State Law or Commission Policy for MIAA & MIL Programs**

When applying to the Commission for program approval, a program sponsor will describe its process for assessing program preconditions. This section outlines the process, by which programs must verify completion of program preconditions, including any attempts by the candidate to meet requirements through equivalency and comparability. Equivalence is defined as the determination that a given set of knowledge, skills, and abilities as reflected in standards have been met through coursework/fieldwork/prior learning experiences as determined by an evaluation process. Comparability is defined as the determination that a candidate has demonstrated the essence of the set of knowledge, skills or abilities required by a particular Commission program standard through another route. While the assessment process is up to the discretion of the program sponsor, the process must be consistent and transparent. For a fuller discussion of CTC requirements concerning Equivalence and Comparability, refer to Comparability of Coursework for Sponsors of Special Education Teacher Preparation Programs, Report to the Governor, Legislature, and Secretary of Education as Required by AB 2226 (Chap. 233, Stats. 2008), January 2010.

The following are non-restrictive examples of options which may be used to verify competency met by the candidate:

- A transcript that provides sufficient information regarding course content, catalog descriptions, course syllabi, or a matrix identifying variations of course offerings across content areas.
- Examination results that verify the competency of the candidate. Examinations may be program-developed and/or nationally standardized.
- 

Options for verification may include all or part of the above categories as a written record to verify and monitor the completion of program requirements.

If a candidate directly presents evidence to the program sponsor for comparability in meeting specific credential requirements, the following guidelines should prevail:

- The candidate should present information based on current, adopted California content standards as appropriate.

- As written agreements are formulated between the candidate and the Program Sponsor, such agreements must be evidence-based as appropriate to the specific, credential standard requested for equivalence.
- Evaluative information presented by the candidate must be in line with the *California Standards for the Teaching Profession*.

### **Completion of the MIAA and MIL Programs**

Approved program sponsors will determine that the candidate has completed the approved program. When the candidate finishes the program, the program sponsor will submit the candidate's recommendation through the Commission's electronic recommendation process.