

Mathematics Teacher Preparation in California: Standards of Quality and Effectiveness for Subject Matter Programs

**A Handbook for
Teacher Educators
&
Program Reviewers**



(Revised November 2009)

Mathematics Teacher Preparation in California: Standards of Quality and Effectiveness for Subject Matter Programs

**Created and Recommended by the
Mathematics Subject Matter Advisory Panel
(2001-2003)**



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Gray Davis, Governor
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California Commission on Teacher Credentialing 2001-03

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Part 1: Introduction to Mathematics Teaching Standards

Standards and Credentials for Teachers of Mathematics: Foreword by the Commission on Teacher Credentialing

One of the purposes of education is to enable students to learn the important subjects of the school curriculum to further their professional goals and to function effectively in work, society, and family life. Each year in California, more than one million students enroll in mathematics classes with teachers who are certified by the California Commission on Teacher Credentialing (CCTC) to teach those classes in public schools. Students who are the future of California and the nation must learn to use mathematics thoughtfully and skillfully. Their ability to do so depends substantially on the quality of teacher preparation in mathematics and mathematics teaching.

The Commission is the agency of California government that certifies the competence of teachers and other professionals who serve in the public schools. As the policy-making body that establishes and maintains standards for the education profession in the state, the California Commission on Teacher Credentialing is concerned with the quality and effectiveness of the preparation of teachers and other school practitioners. On behalf of the education profession and the general public, one of the Commission's most important responsibilities is establishing and implementing strong, effective standards of quality for the preparation and assessment of credential candidates

Teacher candidates in California are required to demonstrate competence in the subject matter they will be authorized to teach. Candidates for the Single Subject Credential have two options available for satisfying this requirement. They can either complete a Commission-approved subject matter preparation program or they can pass the appropriate Commission-adopted subject matter examination(s) (Education Code Sections 44280 and 44310). Because they satisfy the same requirement, these two options are to be as aligned and congruent as possible.

The substance and relevance of the single subject matter program standards and the validity of examination specifications (subject matter requirements) is not permanent, however. The periodic reconsideration of subject matter program standards and the need for periodic validity studies are directly related to one of the fundamental missions of the Commission: to provide a strong assurance that teaching credentials issued by the Commission are awarded to individuals who have the knowledge, skills, and abilities needed to succeed in California public school teaching positions. Best professional practice related to the program standards and the legal defensibility of the examination specifications require that the standards and specifications be reviewed periodically and rewritten, as job requirements and expectations change over time (Ed Code 44225i, j, 44257, 44288).

In the early 1990s the Commission developed and adopted (a) standards for single subject matter preparation programs and, at the same time, (b) specifications for the single subject matter examinations. This work was based on the advice of single subject matter advisory panels and data from validity studies and resulted in program standards and examination specifications (defining the subject matter competence requirement) that were valid and closely aligned with each other. Those standards and specifications were adopted by the Commission in 1992 and are still in use today. They are now being replaced by the newly adopted (2002) subject matter requirements and single subject matter standards.

Establishing high standards for teachers is based, in part, on three major pieces of legislation. In 1988, 1992 and 1998 the Legislature and the governor enacted legislation sponsored by the Commission that strengthened the professional character of the Commission and enhanced its authority to establish rigorous standards for the preparation and assessment of prospective teachers. These reform laws were Senate Bills 148 (1988), 1422 (1992) Bergeson, and 2042 (Alpert/Mazzoni, Chapter 548, Statutes of 1998). As a result, the Commission has taken on new responsibilities for establishing high and acceptable levels of quality in teacher preparation and of competence among beginning teachers. To implement these three statutes, the Commission has developed new standards, subject matter requirements and other policies collaboratively with representatives of post-secondary institutions, teachers and administrators in public schools, and statewide leaders involved in public education.

In the late 1990s, the State Board of Education adopted K-12 student academic content standards in English, mathematics, science, and social science. These new standards direct implications for the subject matter competence requirement of prospective teachers. This was recognized in SB 2042 (Alpert/Mazzoni, Chapter 548, Statutes of 1998), which requires the Commission to ensure that subject matter program standards and examinations are aligned with the K-12 student content standards adopted by the State Board of Education.

The Commission appointed four panels in 1999 (English, mathematics, science, and social science) to begin the first of three phases to meet the SB 2042 mandate for single subject matter programs. The second and third phases will bring all 13 subject matter areas for credentials into alignment with K-12 student content standards by 2005. The first phase single subject matter panels (2001, 2002) spent considerable time to ensure that the new subject matter standards were grounded in, and aligned with, the K-12 student academic content standards.

Standards of Program Quality and Effectiveness

Over the past 15 years the Commission has thoroughly redesigned its policies regarding the preparation of education professionals and the review of preparation programs in colleges and universities. In initiating these reforms, the Commission adopted the following principles regarding the governance of educator preparation programs. The Commission asked the Single Subject Panels to apply these general principles to the creation of standards for subject matter programs in English, mathematics, science and social science.

- (1) *The status of teacher preparation programs in colleges and universities should be determined on the basis of standards that relate to significant aspects of the quality of those programs.*
- (2) *There are many ways in which a teacher preparation program could be excellent.*
- (3) *The curriculum of teacher education plays a central role in a program's quality.*
- (4) *Teacher education programs should prepare candidates to teach the public school curriculum effectively.*
- (5) *In California's public schools, the student population is so diverse that the preparation of educators to teach culturally diverse students cannot be the exclusive responsibility of professional preparation programs in schools of education.*
- (6) *The curriculum of a teacher education program should be based on an explicit statement of purpose and philosophy. An excellent program also includes student services and policies such as advisement services and admission policies.*

- (7) *The Commission is concerned about the high level of attrition among beginning teachers, and has successfully sponsored legislation to improve the conditions in which new teachers work.*
- (8) *The assessment of each student's attainments in a teacher education program is a significant responsibility of the institution that offers the program.*
- (9) *The Commission's standards of program quality allow quality to assume different forms in different environments.*
- (10) *The Commission's standards of program quality are roughly equivalent in breadth and importance..*
- (11) *Whether a particular program fulfills the Commission's standards is a judgment that is made by professionals who have been trained in interpreting the standards.*

The Commission fulfills one of its responsibilities to the public and the profession by adopting and implementing standards of program quality and effectiveness. While assuring the public that educator preparation is excellent, the Commission respects the considered judgments of educational institutions and professional educators and holds educators accountable for excellence. The premises and principles outlined above reflect the Commission's approach to fulfilling its responsibilities under the law.

Standards for Professional Teacher Preparation Programs

The effectiveness of the mathematics curriculum in California schools does not depend entirely on the content knowledge of mathematics teachers. Another critical factor is the teachers' ability to *teach* mathematics. To address the *pedagogical* knowledge and effectiveness of mathematics teachers, the Commission in September 1998 launched an extensive standards and assessment reform that led to the development of new teacher preparation standards. In January 2001, the Commission authorized an extensive field review of the draft standards, and in July a summary and analysis of the field review findings were presented to the Commission. During July and August 2001, the standards were amended, based on field review findings and direction from the Commission, and finally adopted by the Commission in September 2001.

The advisory panel that developed the standards was charged with developing the following three policy documents for review and consideration by the Commission:

- New standards of quality and effectiveness for professional teacher preparation programs.
- Teaching Performance Expectations that would serve as the basis for evaluating the competence of teacher candidates on teaching performance assessments embedded in preparation programs.
- New standards of quality and effectiveness for professional teacher induction programs.

These standards implement the structural changes in the teacher credentialing system that were called for in Senate Bill 2042 (Alpert/Mazzoni, 1998). Three significant changes enacted in this reform legislation are (1) alignment of all teacher preparation standards with the state-adopted academic content standards and performance levels for students and the California Standards for the Teaching Profession (CSTP), (2) the inclusion of a teaching performance assessment in preparation programs, and (3) a required induction period of support and formative assessment for all first and second year teachers.

In addition to these structural and thematic shifts in the Commission's credentialing system and standards, SB 2042 replaced the Professional Clear Credential course requirements in health, mainstreaming and technology with a requirement that essential preparation in these three areas be addressed in preparation and induction standards. Follow-up legislation in 1999 (Duchenev, Chapter 711, Statutes of 1999) required that new standards for preparation and induction programs include preparation for all teachers to teach English learners in mainstream classrooms. The subject matter standards in this handbook have been designed to complement the SB 2042 standards for programs of pedagogical preparation.

Subject Matter Preparation Programs for Prospective Teachers

In California, subject matter preparation programs for prospective teachers are not the same as undergraduate degree programs. Post-secondary institutions govern academic programs that lead to the award of degrees, including baccalaureate degrees in mathematics. The Commission sets standards for academic programs that lead to the issuance of credentials, including the Single Subject Teaching Credential in Mathematics. An applicant for a teaching credential must have earned a bachelor's degree from an accredited institution, but the degree may be in a subject other than the one to appear on the credential. Similarly, degree programs for undergraduate students in mathematics may or may not fulfill the Commission's standards for subject matter preparation. Completing an approved subject matter program that satisfies the standards enables a candidate to qualify for the Single Subject Credential in Mathematics.

Subject Matter Advisory Panels

The Commission asked the Mathematics Subject Matter Advisory Panel to create new standards of program quality and effectiveness that could be used to review and approve subject matter preparation programs. The Commission requested the development of standards that would emphasize the knowledge, skills and perspectives that teachers must have in order to teach mathematics effectively in the public schools.

In January 2001 the executive director appointed subject matter panels in English, mathematics, science, and social science to advise Commission staff on the development of new subject matter program standards and examinations in these subject areas. Each panel consists of:

- Classroom teachers of the subject area,
- Subject area specialists in school districts, county offices of education, and post-secondary institutions,
- Professors in the subject area teaching in subject matter preparation programs,
- Teacher educators,
- Members of relevant professional organizations,
- Members of other relevant committees and advisory panels, and
- A liaison from the California Department of Education.

Eighteen panel members were appointed to the English panel; 17 members appointed to the mathematics panel; 20 appointed to the social science; and 23 appointed to the science panel. The panels began their work in March 2001 with a written "charge" describing their responsibilities in developing the Subject Matter Requirements (SMRs). The SMRs are the subject-specific knowledge, skills, and abilities which specify the content required in Commission-approved subject

matter preparation programs for teacher candidates. The SMRs were approved by the Commission at a meeting on June 6, 2002.

Essential Documents for Panel Use

From their first meeting in March 2001, the subject matter panels used a number of documents as primary resources for their work. The documents listed below were essential for the panels' use in developing the draft program standards that were adopted by the Commission.

- The K-12 Student Academic Content Standards and Frameworks that have been approved by the California State Board of Education (1998-2002).
- The Commission-approved (1992) Standards of Quality and Effectiveness for Subject Matter Programs in English, Mathematics, Science, and Social Science, and Handbooks for Teacher Educators and Program Reviewers in each of the four academic areas (1992).
- The Standards of Program Quality and Effectiveness for the Subject Matter Requirements for the Multiple Subject Teaching Credential (Sept., 2001).
- The Standards for Quality and Effectiveness for Professional Teacher Preparation Programs (Sept., 2001).
- The National Standards for the National Council for Teachers of Mathematics (NCTM), National Council for Teachers of English (NCTE), National Council for Social Science (NCSS), and National Science Teachers Association (NSTA).
- The panels also reviewed several other publications and research articles. Several panel members brought state and national studies and publications for each panels' use.

The State Board of Education adopted K-12 student academic content standards were the seminal documents used by the panels. In the 1992 documents the panels identified six standard areas that were common to each of the four sets of academic standards. This process was instrumental in assisting the panels in identifying the 10 "Standards Common to All" that were developed and applied to all 13 single subject areas. In 2010 the 10 Standards Common to All were revised and replaced with two new Standards Common to All.

The *Subject Matter Requirements for the Multiple Subject Teaching Credential* was also an important document used by the panels. In some cases the multiple subject standards language and the organization of the standards were incorporated by the panels. The standards of the national professional organizations also served as a guide and provided a comprehensive perspective for panel members.

Field Review Survey

Early in August 2002 the draft Single Subject Matter Standards and the Ten "Standards Common to All" were mailed to all deans of education, directors of teacher education, and single subject coordinators at all Commission-accredited four-year institutions in California, learned societies and professional organizations, funded subject matter projects, teacher organizations, school districts, and county offices of education. Over 100 selected K-12 public school teachers and college/university professors were sent the draft standards. The standards were also placed on the Commission's web-site with instructions on how to download the standards and complete the field review survey and how to fax, email, or mail completed surveys to the Commission.

There were 717 responses submitted to the Commission in October, 2002. Over 80% of all responses fell in the "Essential" or "Important" categories. Fewer than 5% of all responses were

scored as “Not Important”, and less than 15% were scored as “Somewhat Important.” The responses were evenly distributed among the five sets of standards.

Over 80% of all responses were from higher education faculty at colleges and universities in California. Over 70% of responses were received from academic departments or faculty in the California State University (CSU) system. Responses were received from all 23 CSU campuses, five University of California campuses, and 14 private or independent institutions. The CSU Academic Senate was instrumental in obtaining strong responses from academic departments in the CSU system.

Consultant staff tallied all responses and listed all comments on a master survey form for each subject matter area. The Single Subject Matter Panels made revisions in the language of certain standards, based on the 2002 field review, and the revised standards were recommended to the Commission for adoption at its December 5, 2002, meeting. The Commission also approved eight technical assistance meetings for spring 2003 and an implementation plan for the new standards.

The Mathematics Teaching Credential

Single Subject Teaching Credentials in Mathematics authorize holders to teach mathematics classes in departmentalized settings. These credentials authorize teaching at any grade level, but departmentalized teaching of mathematics usually occurs in grades seven through twelve. The Commission asked the Mathematics Subject Matter Advisory Panel to recommend new policies to ensure that future teachers of mathematics are prepared to instruct the subjects that are most commonly taught in mathematics classes. In 2000-01, when the advisory panel was established, 30% of all mathematics classes in California public schools were general courses in basic or remedial mathematics for students in grades seven through twelve. The remaining 70% of the classes taught by mathematics teachers were in the following subjects:

Pre-algebra	11% of all mathematics classes
Beginning and Intermediate Algebra	33%
Plane and Solid Geometry	9%
Trigonometry	1%
Pre-calculus and Calculus	3%
Integrated Mathematics	7%
Other Mathematics Subjects	6%

In their deliberations the Advisory Panel expressed concern about a growing shortage of mathematics credential candidates and recommended that the Commission establish a bifurcated structure for mathematics credentials. This proposed structure would continue to offer a mathematics credential with an authorization to teach any departmentalized K-12 mathematics course, but would also offer a new mathematics credential with an authorization limited to teaching courses up through all levels of algebra and geometry, excluding advanced placement courses. This would allow those who majored in applied mathematics fields and thus have deep knowledge of most levels of mathematics, but without a background in the highly-advanced concepts required of pure mathematics majors and required for the full mathematics authorization, to qualify for a limited mathematics authorization. In 2003, based on the Advisory Panel’s recommendation, California introduced a new credential in Foundational Mathematics. **The standards and other policies in this document are footnoted where appropriate to differentiate those standards and requirements that do not apply to the Foundational-level credential.** The standards and other

policies for the Foundational Mathematics Credential are designed to prepare teachers for basic and remedial classes in mathematics as well as algebra, geometry, and integrated mathematics courses with equivalent content. The entire set of standards and other policies in this manual pertain to the full Single Subject Credential in Mathematics, and are designed to prepare teachers for basic and remedial classes in mathematics as well as all of the more advanced courses listed above.

Alignment of Program Standards and Performance Assessments

The Teacher Preparation and Licensing Act of 1970 (Ryan Act) established the requirement that candidates for teaching credentials verify their knowledge of the subjects they intend to teach. Candidates for teaching credentials may satisfy the subject matter requirement by completing approved subject matter programs or passing subject matter examinations that have been adopted by the Commission. In 1998 Senate Bill 2042 required that subject matter programs and examinations for prospective teachers be aligned with K-12 student standards and frameworks.

To achieve this alignment and congruence in mathematics, the Commission asked the mathematics Subject Matter Advisory Panel to develop subject matter requirements that would be consistent in scope and content with the K-12 standards and frameworks. Following extensive research and review, the Commission adopted a detailed set of *Subject Matter Requirements for Prospective Teachers of Mathematics*, which follow the standards in this handbook. College and university faculty and administrators are urged to examine these requirements as a source of information about content that is essential to include in subject matter preparation programs.

The Commission sought to align the subject matter requirements with the program standards in each subject area. Each subject matter advisory panel is asked to develop standards and subject matter requirements that are as congruent with each other as possible, to maximize the equivalence between credentials that are earned by completing programs and ones that are earned by passing examinations. Standards and examinations were developed from the same set of subject matter requirements.

New Subject Matter Assessments

The Commission has used a variety of assessments to satisfy the examination option for various subject areas. In the early 1990s, the Commission developed and adopted (a) standards for subject matter preparation programs and, at the same time, (b) specifications for the subject matter examinations. The validity of the subject matter competence requirement (i.e., program standards and examination specifications) is not permanent, however. The need for periodic validity studies of the subject matter requirement is directly related to one of the Commission's most fundamental missions: to provide a strong assurance that teaching credentials are awarded to individuals who have learned the most important knowledge, skills, and abilities that are actually needed in order to succeed in California public school teaching positions.

In the late 1990s, the State Board of Education adopted K-12 student content standards in English, mathematics, science, and social science. Beginning in early 2001, the Commission began the process of developing assessments that were aligned with these new standards. In the spring of 2002, the Commission contracted with National Evaluation Systems, Inc. (NES®) to implement a new examination program called the California Subject Examinations for Teachers (CSET). In the four subject areas, multiple-choice and constructed-response items were drafted based on the subject

matter requirements, and reviewed and revised as needed by both the Bias Review Committee and the appropriate subject matter advisory panel.

The CSET for English, mathematics, science, and social science were first administered in January 2003, and by June 2003, fully replaced the SSAT and Praxis II examinations as the new subject matter examinations in these areas. From January through June 2003, teacher candidates in these subject areas were allowed to use either the new CSET or the combination of appropriate SSAT and Praxis II examinations.

Overview of the Mathematics Standards Handbook

This introduction to the handbook concludes with a statement by the Mathematics Advisory Panel regarding mathematics teaching and teacher preparation in California. Part 2 of the handbook includes the sixteen standards as well as the Subject Matter Requirements for Prospective Teachers of Mathematics. Finally, Part 3 provides information about implementation of the new standards in California colleges and universities.

Contributions of the Mathematics Advisory Panel

The Commission on Teacher Credentialing is indebted to the Mathematics Teacher Subject Matter Advisory Panel for the successful creation of *Standards of Program Quality and Effectiveness for the Subject Matter Preparation of Prospective Teachers of Mathematics*. The Commission believes strongly that the standards in this handbook will improve the teaching and learning of mathematics in California's public schools.

Request for Assistance from Handbook Users

The Commission periodically reviews its policies, in part on the basis of responses from colleges, universities, school districts, county offices, professional organizations and individual professionals. The Commission welcomes all comments and questions about the standards and other policies in this handbook, which should be addressed to:

California Commission on Teacher Credentialing
Professional Services Division
1900 Capitol Avenue
Sacramento, California 95811

Mathematics Teaching and Teacher Preparation: Introduction by the Mathematics Advisory Panel

With the adoption of the California Academic Standards for Mathematics by the State Board of Education in 1997, it became apparent that teachers of mathematics must be better prepared to teach to these higher standards. The Standards for quality and effectiveness for Mathematics Subject Matter Programs is a direct outgrowth of the need for better prepared mathematics teachers.

These Standards are divided into three parts: Common Standards, Standards that are common to all disciplines; Program Standards, standards that apply to the mathematics program as a whole; and Subject Matter Requirements (SMR's), those subject areas that are required parts of the program.

Common Standards - The Common Standards deal with philosophy and purpose, diversity and equity, program coordination, advisement and support, assessment, program review, literacy, and varied teaching strategies. Every subject matter program must give attention to each of these standards specific to the subject matter program.

Program Standards - Program Standards for Mathematics set forth the required subjects of study for problem solving, communication, reasoning, mathematical connections, and delivery of instruction. These standards are intended to identify the qualities of mathematical preparation that are particularly important for prospective teachers. Because these are program standards, it is not necessary that every course in the program meets every standard. However, throughout the breadth of the program, every program standard must be met.

The program standards are not intended to require mathematics departments to create a completely separate set of courses for prospective secondary mathematics teachers from those courses used to satisfy a mathematics degree. In most cases the math courses under a previously approved subject matter program already address most of the standards. Nevertheless, mathematics departments should take this opportunity to consider ways to improve their programs so that prospective teachers are better able to apply what they have learned, and how they have learned, to the profession of teaching mathematics at the secondary level. As a result, departments may initiate revisions of existing courses or create new courses or experiences for prospective teachers in order to meet the full measure of the program standards.

Subject Matter Requirements - The Subject Matter Requirements are based on the concept that those who teach to the *K-12 California Academic Content Standards for Mathematics* should understand the mathematics that is contained therein from an advanced viewpoint. Hence, the SMR's are divided into six domains that link to the content standards: algebra, geometry, number theory, probability and statistics, trigonometry and calculus, and history of mathematics. Under each domain is a list of elements that must be included in the program. These domain elements are prescriptive, but in no way are they exhaustive. Each program will likely provide candidates with much more breadth and depth for developing an "advanced perspective" than is represented by the set of domain elements. The elements form only a baseline on which a complete mathematics program may be built.

The appendix to the SMR's provides one vision of the depth of coverage of the SMR's that will contribute to a prospective teacher developing an advanced perspective on the mathematics they are

expected to teach. The lack of additional prescription should be viewed as an invitation to mathematics departments to develop programs that build on the strengths of faculty and existing programs as well as local values and conditions.

Process

The process to develop these new standards for quality took 18 months. Beginning with the California Academic Standards, subject matter requirements suitable for a beginning teacher were developed and submitted to field review both for bias and for appropriateness. On the basis of the review, the SMR's were revised and submitted to the CCTC for approval. At the same time, a plan for the new testing program was developed by the panel and submitted with the SMR's to a test developer. The text developer produced some sample items which were critiqued by the panel. With a more comprehensive view, the developer was able to complete a bank of test items which was reviewed by the panel on an individual item basis.

Parallel to the field reviews of the SMR's, the Panel completed work on the program standards which were also submitted for field review. Following the review, changes were made and the package was submitted to the CTC for approval.

The Advisory Panel Recommendation - As in the past, prospective teachers of mathematics can obtain "subject matter competence" by examination. The examination route is based only on the SMR's and the underlying K-12 academic content standards in mathematics. There is no assessment of the further advanced view of school mathematics or of the rich experiences in the discipline that will be offered by approved subject matter programs. In other words, candidates who chose the examination route as a means to bypass earning a degree in mathematics will miss much of what subject matter experts deem critical for a well-prepared teacher of secondary mathematics. Therefore, it is imperative that subject matter programs and the associated mathematics degrees are maintained and promoted as the preferred method of preparation for middle school and high school mathematics teaching. Candidates who choose the program route will be better prepared to provide the leadership in mathematics education that their schools and districts require.

Part 2: Standards of Program Quality in Mathematics

Definitions of Key Terms

California state law authorizes the California Commission on Teacher Credentialing to set standards and requirements for preparation programs (Ed Code 44225a, i, j, 44310, 44311).

Preconditions

A precondition is a requirement for initial and continued program approval. Unlike standards, preconditions specify requirements for program compliance, not program quality. The Commission determines whether a program complies with the adopted preconditions on the basis of a program document provided by the college or university. In the program review sequence, a program that meets all preconditions is eligible for a more intensive review to determine if the program's quality satisfies the Commission's standards. Preconditions for the approval of subject matter programs in mathematics are on following pages.

Standards

Standards are statements of program quality adopted by the Commission on Teacher Credentialing to describe acceptable levels of quality in programs of subject matter study offered by regionally-accredited colleges and universities that award baccalaureate degrees. Each standard is elaborated by Program Guidance for that standard. Programs must meet all of the applicable standards for both initial and continuing approval of a subject matter program by the Commission. The Commission determines whether a program satisfies a standard on the basis of an intensive review of all available information provided by the program sponsor related to the standard.

Program Guidance

Program guidance is provided for each standard to help institutions in developing programs that meet the standards, and are also used by program review panels in judging the quality of a program in relation to a given standard. Within the overall scope of a standard, Program Guidance identifies what the Commission believes are the important dimensions of program quality with respect to each standard. In determining whether a program fulfills a given standard, the review panel considers the information provided by the program in response to each statement of that standard. When the review panel finds that a program has met each standard, the program is then recommended to the Commission for approval.

Preconditions for the Approval of Subject Matter Programs in Mathematics

To be approved by the Commission, a Subject Matter Program in Mathematics must comply with the following preconditions.

- (1) Each program of subject matter preparation for the Single Subject Teaching Credential in Mathematics shall include (a) a minimum of 30 semester units (or 45 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides extended study of the subject. These two requirements are elaborated in Preconditions 2 and 3.
- (2) The core of the program shall include coursework in subjects commonly taught in departmentalized classes of mathematics and related subjects in the California public schools such as algebra (or demonstrated proficiency), geometry, number theory, calculus, history of mathematics, and statistics and probability.
- (3) Extended studies (breadth, depth, perspective, concentrations) in the program shall be designed to supplement the core of the program.

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include the course titles, unit designations, catalog descriptions and syllabi of all courses in the program that are used to meet the standards. Program documents must also include a matrix that identifies which courses meet which subject matter requirements.

Institutions may determine whether the standards are addressed through one or more courses for each commonly taught subject or courses offering integrated study of these subjects. Institutions may also define the program in terms of required or elective coursework. However, elective options must all meet the standards. Coursework offered by any appropriate department(s) of a regionally accredited institution may satisfy the preconditions and standards in this handbook. Programs may use general education courses in meeting the standards.

Preconditions for the Approval of Subject Matter Programs in Foundational Mathematics

To be approved by the Commission, a Subject Matter Program in Foundational Mathematics must comply with the following preconditions.

- (1) Each program of subject matter preparation for the Single Subject Teaching Credential in Foundational Mathematics shall include (a) a minimum of 20 semester units (or 30 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 12 semester units (or 18 quarter units) of coursework that provides extended study in applications of the subject. These two requirements are elaborated in Preconditions 2 and 3.
- (2) The core of the program shall include coursework in subjects commonly taught in departmentalized classes of mathematics and related subjects in the California public schools such as algebra (or demonstrated proficiency), geometry, number theory, and statistics and probability.
- (3) Extended studies (breadth, depth, perspective, concentrations) in the program shall be designed to supplement the core of the program with coursework which has a strong mathematical basis (e.g. physics, engineering, computer programming).

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include the course titles, unit designations, catalog descriptions and syllabi of all courses in the program that are used to meet the standards. Program documents must also include a matrix that identifies which courses meet which subject matter requirements.

Institutions may determine whether the standards are addressed through one or more courses for each commonly taught subject or courses offering integrated study of these subjects. Institutions may also define the program in terms of required or elective coursework. However, elective options must all meet the standards. Coursework offered by any appropriate department(s) of a regionally accredited institution may satisfy the preconditions and standards in this handbook. Programs may use general education courses in meeting the standards.

Standards of Program Quality and Effectiveness

Category I: Standards Common to All Single Subject Matter Preparation Programs

(These standards are required for both full Single Subject and Foundational-level Mathematics programs.)

Standard 1: Program Design

Subject matter programs are based on an explicit statement expressing the purpose, design, and expected outcomes of the program. The program curriculum builds on the K-12 State-adopted academic content standards, with student outcomes and assessments aligned to the subject matter requirements. The program provides prospective teachers with conceptual knowledge of the subject matter, develops academic literacy and discipline-based fluency, addresses issues of equity and diversity, and exposes prospective teachers to a variety of learning experiences appropriate for the discipline.

Standard 2: Program Resources and Support

The program sponsor allocates resources to support effective program coordination, which includes advising students, facilitating collaboration among stakeholders, and overseeing program review. Ongoing review processes use assessments of the prospective teachers and a variety of data such as input from stakeholders and other appropriate measurements for review and evaluation of the subject matter program.

Category II: Mathematics Subject Matter Program Standards

Standard 3: Required Subjects of Study

In the program, each prospective teacher studies and learns advanced mathematics that incorporates the Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve (1997) and the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999). The curriculum of the program addresses the *Subject Matter Requirements* and standards of program quality as set forth in this document.¹

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

- *Required coursework includes the following major subject areas of study: algebra, geometry, number theory, calculus, history of mathematics, and statistics and probability. This coursework also incorporates the content of the student academic content standards from an advanced viewpoint (see *Attachment to Standard 11: Required Subjects of Study* page 18). Furthermore, infused in required coursework are connections to the middle school and high school curriculum.
- Required coursework exposes underlying mathematical reasoning, explores connections among the branches of mathematics, and provides opportunities for problem solving and mathematical communication.
- Required courses are applicable to the requirements for a major in mathematics. Remedial classes and other studies normally completed in K-12 schools are not counted in satisfaction of the required subjects of study.
- The institution that sponsors the program determines, establishes and implements a standard of minimum scholarship for coursework in the program.
- Required coursework includes work in computer science and/or related mathematics such as: 1) discrete structures (sets, logic, relations and functions) and their application in the design of data structures and programming; 2) design and analysis of algorithms including the use of recursion and combinations; and, 3) use of the computer applications and other technologies to solve problems.

*Calculus and history of mathematics are not required subjects of study for the foundational-level credential.

¹ The Subject Matter Requirements are complemented by the **Attachment to Standard 11**, starting on page 32

Standard 4: Problem Solving

In the program, prospective teachers of mathematics develop effective strategies for solving problems both within the discipline of mathematics and in applied settings that include non-routine situations. Problem-solving challenges occur throughout the program of subject matter preparation in mathematics. Through coursework in the program, prospective teachers develop a sense of inquiry and perseverance in solving problems.

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

In the program, each prospective teacher learns and demonstrates the ability to:

- Place mathematical problems in context and explore their relationship with other problems.
- Solve mathematical problems in more than one way when possible.
- Generalize mathematical problems in more than one way when possible.
- Use appropriate technologies to conduct investigations and solve problems.

Standard 5: Mathematics as Communication

In the program, prospective teachers learn to communicate their thinking clearly and coherently to others using appropriate language, symbols and technologies. Prospective teachers develop communication skills in conjunction with mathematical literacy in each major component of a subject matter program.

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

In the program, each prospective teacher learns and demonstrates the ability to:

- Articulate mathematical ideas verbally and in writing, using appropriate terminology.
- Where appropriate present mathematical explanations suitable to a variety of grade levels.
- Present mathematical information in various forms, including but not limited to models, charts, graphs, tables, figures, and equations.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use clarifying and extending questions to learn and to communicate mathematical ideas.
- Use appropriate technologies to present mathematical ideas and concepts.

Standard 6: Reasoning

In the program, prospective teachers of mathematics learn to understand that reasoning is fundamental to knowing and doing mathematics. Reasoning and proof accompany all mathematical activities in the program.

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

In the program, each prospective teacher learns and demonstrates the ability to:

- Formulate and test conjectures using inductive reasoning, construct counter-examples, make valid deductive arguments, and judge the validity of mathematical arguments in each content domain of the subject matter requirements.
- Present informal and formal proofs in oral and written formats in each content domain of the subject matter requirements.

Standard 7: Mathematical Connections

In the program, prospective teachers of mathematics develop a view of mathematics as an integrated whole, seeing connections across different mathematical content areas. Relationships among mathematical subjects and applications are a consistent theme of the subject matter program's curriculum².

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

In the program, each prospective teacher learns and demonstrates the ability to:

- Illustrate, when possible, abstract mathematical concepts using applications.
- Investigate ways mathematical topics are inter-related.
- Apply mathematical thinking and modeling to solve problems that arise in other disciplines.
- Recognize how a given mathematical model can represent a variety of situations.
- Create a variety of models to represent a single situation.
- Understand the interconnectedness of topics in mathematics from an historical perspective.

² The Subject Matter Requirements are complemented by the **Attachment to Standard 11**, starting on page 32.

Standard 8: Delivery of Instruction

In the program, faculty use multiple instructional strategies, activities and materials that are appropriate for effective mathematics instruction.

The following statements no longer require a direct response but should be used for guidance in responding to the standards directly. Each statement of the standard should be responded to instead, by providing a brief description, a few examples and evidence citations for how the program meets the standard. Please limit the total response to the standard to 1-2 pages.

Coursework in the program:

- Is taught in a way that fosters conceptual understanding as well as procedural knowledge.
- Incorporates a variety of instructional formats including but not limited to direct instruction, collaborative groups, individual exploration, peer instruction, and whole class discussion led by students.
- Provides for learning mathematics in different modalities, e.g., visual, auditory, and kinesthetic.
- Develops and reinforces mathematical skills and concepts through open-ended activities.
- Uses a variety of appropriate technologies.
- Includes approaches that are appropriate for use at a variety of grade levels.

Attachment to Standard 3: Required Subjects of Study

The main purpose of the Subject Matter Requirements (SMRs) is to provide a guideline for the education of prospective mathematics teachers so that they will be well equipped to teach to the state-adopted Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve (1997), and that they have a mathematical understanding and proficiency beyond those Standards. Taken at face value, the SMRs define a minimum core of skills, abilities, and understandings for all candidates of the Single Subject Teaching Credential in Mathematics. Ideally, teacher candidates develop an advanced viewpoint of the content areas represented in this core. The intent of this attachment is to give a sense of the mathematical context in which such advanced viewpoints can be developed. The attachment provides examples and ideas for this development, and is not intended to be prescriptive. While some of these examples may seem obvious to a professor of mathematics, many mathematics majors do not make the connections. Therefore, these ideas are important for prospective teachers.

It is important to note three principles that guided the development of the SMRs:

- a) mathematical reasoning is central to mathematical understanding;
- b) mathematics requires knowledge that is connected and integrated; and
- c) college faculty are central to shaping the curriculum of subject matter programs.

First, the emphasis on mathematical reasoning amplifies what is already clearly enunciated in a critical passage of the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; Framework):

From kindergarten through grade 7, these [content] standards have impressed on the students the importance of logical reasoning in mathematics. Starting with grade 8, students should be ready for the basic message that logical reasoning is the underpinning of all of mathematics. In other words, every assertion can be justified by logical deduction from previously known facts. Students should begin to learn to prove every statement that they make. Every textbook or mathematics lesson should strive to convey this message, and to convey it well. (p. 154)

In order for such a vision of mathematics education to materialize, teachers themselves need to be well versed in writing proofs and explaining them. For this reason, the SMRs emphasize logical explanations, and formal and informal proofs. Explanations and proofs also underscore the fact that logical arguments occur not only in Euclidean geometry but everywhere.

A proof is a logical explanation of why a statement holds. It need not have any particular form, and the emphasis should be on the student understanding why a result holds. Written proofs in textbooks may serve as a model for exposition, but never as a model for the discovery of a proof. Proofs are usually found by painstaking trials and errors, and almost never in the logical sequence of steps laid out in written proofs. It should be emphasized that it is the logical correctness of a proof that is important, not the literary polish of the presentation of the proof. The common complaint that geometry proofs in a real classroom have become a ritual divorced from mathematics would disappear if teachers are made more aware of the need to pay attention to mathematical substance rather than minute details of the write-up of a proof. A correct proof

can be legitimately presented in many ways (e.g., two-column format, paragraph format, flow-chart format). No one format is inherently superior to any other.

Second, the integration of subject matter is implied in more than a few of the standards. Although the SMRs are divided into separate content domains (e.g., algebra, geometry) such a division is more for the convenience of communication rather than an advocacy for a rigid separation of mathematical instruction. For example, prospective teachers should be able to analyze and solve polynomial equations using the roots of unity. This statement assumes that the prospective teacher understands De Moivre's Theorem (SMR 5.1e) and basic properties of regular polygons. In this case, algebra, trigonometry, and geometry are completely intermingled. As another example, prospective teachers need to be able to teach the graphing of polynomials, but simple facts about such graphs (e.g., that the graph of an n th degree polynomial has at most $n-1$ "peaks" and "valleys") are not accessible without the use of calculus.

Third, the SMRs are not prescriptive about curriculum or pedagogy. There is plenty of room for the creative and informed judgements of faculty to direct the education of teachers of mathematics. For example, although it is not included in the SMRs, faculty may choose to present the derivation of the cubic formula for the purpose of deepening teachers' understanding and appreciation of the quadratic formula. Similarly, some faculty may view SMR 1.3c, which deals with properties of the logarithm function, as an implicit invitation to go into the origin of the logarithm. Napier's invention of logarithms in the 1600s was the device which, in the word of the French mathematician-astronomer Laplace, "by shortening the labors, doubled the life of the astronomer." When teachers understand this utility, and the parallels of the discovery of logarithms with the discovery and development of computing technologies, they are much better equipped to motivate students' understanding of such mathematical topics.

The following sections provide some ideas and examples for developing an advanced viewpoint, particularly about the importance of mathematical reasoning and connections, through the main subject areas of the SMRs.

Algebra

Mathematical reasoning

Prospective teachers' understanding of the three fields they use most often – rational, real, and complex numbers – should include what it means for rational and real numbers to be ordered fields, and why complex numbers cannot be ordered. Inequalities make sense in real numbers, because they are ordered. However, prospective teachers should understand that although inequalities do not make sense in complex numbers, equations have a fuller role with them, because every polynomial equation with real or complex coefficients can be completely solved in complex numbers by the Fundamental Theorem of Algebra (SMR 1.1c, 1.2c).

Implicit in SMR 1.2a, which calls for a proof of why the graph of a linear inequality is a half plane, is the need for a proof of the fact that the graph of a linear function is a straight line. The latter proof requires the use of basic properties of similar triangles.

The proof of the result that the roots of real polynomials come in complex conjugate pairs (SMR 1.2b) allows one to see how to make use of the Fundamental Theorem of Algebra in a nontrivial

way. In the process, one gains a better understanding of both the Fundamental Theorem of Algebra and the Quadratic Formula.

The rational root theorem for polynomials with integer coefficients (SMR 1.2b) is one that students and textbooks often mistake as a recipe for locating all the roots of such a polynomial. By reviewing the proof carefully, a prospective teacher is likely to understand the full meaning of this theorem.

The Binomial Theorem (SMR 1.2b) occupies a place of honor in algebra and has important connections in other areas of mathematics. Prospective teachers should be able to understand one of its most accessible proofs, and thereby learn a substantive application of mathematical induction.

Connections

Although the SMRs are organized into discrete content domains (e.g., algebra or calculus), prospective teachers should learn that these domains cannot be rigidly separated. For example, the importance of the exponential function (SMR 1.3c) stems primarily from the fact that it is the unique solution of the differential equation $f'(x) = f(x)$ with the initial condition $f(0) = 1$ (SMR 5.3f). It should be emphasized that it is because of this differential equation that the exponential function e^x (exp x) shows up in the growth and decay problems of algebra textbooks.

The fundamental difference between polynomial functions and both exponential and logarithmic functions should be emphasized (SMR 1.3b, c). The overriding concern with a polynomial is to locate its roots and the roots of its derivative (to get the x -intercepts as well as the "peaks" and "valleys" of its graph). For exponential and logarithmic functions, however, such a concern does not exist because $\log x$ has exactly one root whereas $\exp x$ has no root at all. Moreover, both are strictly increasing functions; their graphs have no "peaks" or "valleys." Therefore our interests in the latter functions are different in kind. Our interests in the exponential and logarithmic functions are that $\log x$ converts multiplication into addition [i.e., $\log(ab) = \log a + \log b$] while $\exp x$ does the opposite [i.e., $\exp(a+b) = (\exp a)(\exp b)$], and the fact that they are inverses to each other [i.e., $\log(\exp x) = x$ for all x and $\exp(\log y) = y$ for all positive y]. The algebraic properties of $\log x$ account for its historical importance as a computational aid (logarithm tables). Analytically, it is the fact that $\exp x$ is the solution of $f'(x) = f(x)$, as discussed above, and that $\log x$ is the function that has derivative $1/x$ and satisfies $\log 1 = 0$. The trigonometric functions are important for yet a different reason: periodicity (SMR 5.1c). Many natural phenomena are periodic, and their modeling would require the trigonometric functions. Such a conceptual understanding of these three classes of functions is indispensable to helping teachers make sense of the functions they see almost daily in algebra classes.

Although the topic of rationalizing denominators is not one that is seen as essential, it is one for which a strong connection can be made with ideas from an advanced perspective. One example that shows how rationalizing denominators is related to more advanced ideas is the

“rationalizing” of the denominator of $\frac{1}{4^{\frac{1}{3}} - 2(2^{\frac{1}{3}}) + 2}$, which is to find a polynomial in $2^{\frac{1}{3}}$ with

rational coefficients so that multiplying the denominator $4^{\frac{1}{3}} - 2(2^{\frac{1}{3}}) + 2$ by this polynomial

equals a rational number. Let $x = 2^{\frac{1}{3}}$, then the denominator becomes $x^2 - 2x + 2$. In the polynomial ring, $\mathbb{Q}[x]$ (where \mathbb{Q} is the field of rational numbers), the polynomials $x^3 - 2$ and $x^2 - 2x + 2$ are relatively prime and therefore, by the Euclidean algorithm, there are polynomials $p(x)$ and $q(x)$ in $\mathbb{Q}[x]$ so that $p(x)(x^2 - 2x + 2) + q(x)(x^3 - 2) = 1$. Letting $x = 2^{\frac{1}{3}}$ gives $p(2^{\frac{1}{3}})(4^{\frac{1}{3}} - 2(2^{\frac{1}{3}}) + 2) = 1$. It turns out that $p(x) = \frac{1}{10}(x^2 + 3x + 4)$, so that multiplying the

numerator and denominator of $\frac{1}{4^{\frac{1}{3}} - 2(2^{\frac{1}{3}}) + 2}$ by $p(2^{\frac{1}{3}}) = \frac{1}{10}(4^{\frac{1}{3}} + 3(2^{\frac{1}{3}}) + 4)$ leads to

$$\frac{1}{4^{\frac{1}{3}} - 2(2^{\frac{1}{3}}) + 2} = \frac{1}{10} (4^{\frac{1}{3}} + 3(2^{\frac{1}{3}}) + 4).$$

Engaging in this example will help candidates to make a good connection between topics that they studied in their abstract algebra course and ideas related to the high school curriculum.

Geometry

Mathematical Reasoning

The great challenge in a college geometry course for prospective teachers is teaching fluency with informal and formal proofs of geometric theorems in general and theorems in Euclidean geometry (SMR 2.2) in particular. There is a thorough discussion of this issue in Chapter 3 of the 1999 Framework (pp. 162-7; see also Appendix D on pp. 279-296). The following are key points:

- (a) One cannot learn how to prove theorems in geometry without any geometric intuition. One way to acquire such an intuition is to perform constructions with a ruler and compass, and to examine many models of standard solids (e.g., cubes, cones, cylinders).
- (b) An introductory college geometry course should start from the beginning. One way to gain the confidence of prospective teachers is not to force them to write any proofs until they have been shown many nontrivial proofs of interesting theorems (see Appendix D of the 1999 Framework). Begin slowly, allowing them to imitate some standard proofs before they venture forth on their own. This is analogous to the method of teaching people how to speak a foreign language whereby you have them listen to the language for many hours before asking them to try to speak it.
- (c) In middle and high school geometry as well as college-level geometry courses, one should de-emphasize the proofs of simple theorems that come near the beginning of the axiomatic development. The proofs of such theorems are harder to learn than those of theorems that follow, and this is true not only for beginners but also for professional mathematicians as well. These proofs also tend to be tedious and uninspiring. One way to acquaint prospective teachers with the proofs of more substantive theorems as soon as possible is to adopt the method of "local

axiomatics," which is to list the facts one needs for a particular proof, and then proceed to construct the proof on the basis of these facts. This approach mirrors the axiomatic method because, in effect, these facts are the "axioms" in this particular setting (see the examples in Appendix D of the 1999 Framework).

Connections

The historical importance of the parallel postulate, not just in geometry but in all of mathematics up to the nineteenth century, should be thoroughly discussed (SMR 2.1a, b). In middle and high school geometry textbooks, this postulate is stated (if it is stated at all) as "through a point not on a given line, there is one and only one line parallel to the given line." The correct formulation replaces the phrase "there is one and only one" with "there is at most one." In other words, while the existence of the parallel line can be proved, the uniqueness must be assumed. This then gives a natural setting to introduce the concept of "uniqueness," which is a difficult concept for many students. In this context, an informal discussion of the counterparts of the parallel postulate in spherical and hyperbolic geometry (SMR 2.1b) will likely clarify the situation.

The deduction of the parallel postulate from the assumption that "every triangle has an angle sum of 180° " is somewhat more sophisticated than most of the theorems in plane Euclidean geometry, but when done carefully it can be immensely rewarding (SMR 2.1a).

Although the notion of area will be defined using the Riemann integral in the context of calculus (SMR 5.4d), it is essential for the teaching of middle and high school geometry that a basic definition of area be provided for plane geometric figures. From this definition, a prospective teacher should be able to derive the area formulas for regular polygons, and many other plane geometric figures.

The theorem that every polygon can be triangulated into non-overlapping triangles allows the areas of polygons to be calculated once the areas of the triangles are known (SMR 2.2c). There is, however, no analogous theorem for the volume of a general polyhedron (SMR 2.3b). This is because it can be proved (using advanced techniques) that there is no corresponding elementary algorithm to compute the volume of a general (non-regular) tetrahedron from the volume of a cube. Although the proof of this theorem is too difficult for an introductory course, prospective teachers need to know this fact to be able to explain to their students why all volume formulas (except that of a rectangular prism) require the use of calculus or equivalent limit arguments. However, from a basic definition of volume, with the use of informal arguments and Cavalieri's Principle, the volumes of prisms, pyramids, cones, cylinders, and spheres can be informally derived. Moreover, teachers should be aware that formally, the coefficient $1/3$ in the volume formulas of cones and pyramids comes from integrating x^2 (SMR 5.4d).

A key reason for introducing coordinates and discussing geometric transformations (SMR 2.4a, b) is to be able to clarify the concepts of congruence and similarity, not just for triangles or polygons, but for all plane and space figures. In other words, one defines two such figures to be *congruent* if one is the image of the other under an isometry, and defines them to be *similar* if one is the image of the other under an isometry followed by a dilation. Then it can be shown that when the figures are polygons, these concepts coincide with those of the equality of angles and proportionality of sides.

Number Theory

Mathematical Reasoning

The well known divisibility rules for division by 3, 4, 5, 8, or 9 are usually stated and used in middle and high school textbooks but not often explained. It is imperative that prospective teachers understand the simple proofs of these rules (SMR 3.1a).

From the point of view of middle and high school mathematics, there are at least two aspects of the Fundamental Theorem of Arithmetic that are noteworthy. First, a completely correct proof of the existence of a prime decomposition for whole numbers requires the use of complete induction (and this gives an important example of a different application of mathematical induction). Second, whereas in middle and high school mathematics only the existence part of the theorem is used, one discovers that in fact it is the uniqueness of the prime decomposition that is important and difficult to prove. Experience shows that this particular uniqueness statement - more so than the uniqueness in the parallel postulate or the uniqueness of the remainder in the division algorithm - is elusive to beginners. The uniqueness is an essential aspect of the Fundamental Theorem of Arithmetic; otherwise, the proof of the irrationality of $\sqrt{5}$ (or any whole number not a perfect square) or why every fraction is equivalent to a unique fraction in lowest terms would be meaningless.

Connections

The Euclidean algorithm (SMR 3.1c) requires a strong understanding of the division algorithm, including a clear conceptualization of a remainder, and thus the uniqueness of the remainder in the division algorithm. This is another area in which the content domains merge. Prospective teachers should understand both the division algorithm and the Euclidean algorithm for polynomials with real coefficients, and the relationship to the results in number theory.

Calculus*

Mathematical Reasoning

One should emphasize that the sine and cosine addition theorems are the defining theorems of trigonometry (SMR 5.1b). Indeed, it can be proved that sine and cosine are the only differentiated functions satisfying the addition theorems and the condition that $\sin 0 = 0$ and $\cos 0 = 1$. Moreover, every trigonometric identity is a consequence of these addition theorems, and the identity that $\sin^2 x + \cos^2 x = 1$. Thus the latter identity and the addition theorems are the foundation of trigonometry. This fact gives structure to the subject, and should be clearly understood by each prospective teacher.

In the teaching of calculus, it would be inappropriate to insist on epsilon-delta proofs, but it would be equally inappropriate to eliminate such proofs altogether. Therefore, SMR 5.2 requires that at least the correct definition of limit be provided and applied in a restricted way. This can be

accomplished by proving the continuity of quadratic polynomials using epsilon-delta. One benefit of this insistence on a minimal amount of rigor is to expose prospective teachers to the fallacy of the common perception that the continuity of $f(x)$ means "a small change in x produces a small change in $f(x)$." For instance, if this were the case, should not a change in x to the order of $1/10000$ produce a "small" change in $f(x)$? The answer is, of course, no, because if $f(x) = 10^9 x$, then a change in x of $1/10000$ produces a change of 100000 in $f(x)$. Thus, one can see why precision in mathematics (such as that found in the tortuous definition of continuity) is necessary. Not insisting on precise proofs on the most common differentiation formulas is likely to invite some abuse. For example, the usual proof "from the product rule of differentiation, one can prove the quotient rule" is a common pitfall that should be avoided, especially in the context of middle and high school mathematics. The putative proof goes as follows: because $f(x)(1/f(x)) = 1$, differentiating both sides and applying the product rule on the right side of the formula gives $f'(x)(1/f(x)) + f(x)(1/f(x))' = 0$, from which it follows that $(1/f(x))' = -f'(x)/[f(x)]^2$. Once this is known, another application of the product rule to $g(x)(1/f(x))$ gives the usual quotient rule for $g(x)/f(x)$. This is the "proof" of the quotient rule. The fallacy of the preceding argument lies in the fact that until one knows $1/f(x)$ is differentiated one cannot apply the product rule to $f(x)(1/f(x))$. Of course, when one tries to prove the differentiation of $1/f(x)$, the result is the usual messy proof of the quotient rule. What can be claimed is that the above method gives a mnemonic device to remember the quotient rule. Such a statement, when so carefully phrased, has pedagogical value in a calculus classroom, but by no means should one convey the misconception that the product rule proves the quotient rule. Similar comments apply to the differentiation of the square root of a function or, in fact, of any rational power of a function.

The calculus SMRs require the proofs of few theorems, one of which is the proof of the Fundamental Theorem of Calculus (SMR 5.4c). Intended by this SMR is a proof that assumes the basic properties of continuous functions and the integral (e.g., that a continuous function attains a maximum and a minimum on a closed interval, that the integral is linear in the integrand, and that the integral of positive functions is positive). The reason prospective teachers should know this proof is not only that the Fundamental Theorem is truly fundamental (and why this is so should, of course, be carefully explained), but also that this proof is very instructive.

Connections

Both finite and infinite geometric series are important because they appear frequently (SMR 5.5a). In particular, one aspect of infinite geometric series deserves comment, namely the fact that the formal way of summing a geometric series gives rise to the expression of a repeating decimal as a fraction. This mechanism should be conducted carefully as it is often presented incorrectly in middle and high school textbooks. One reason for mentioning the convergence of infinite geometric series (SMR 5.5b) is to make sense of infinite decimals: an infinite decimal is merely a shorthand notation for a particular kind of infinite series. For Taylor series (SMR 5.5c), candidates should know at least the formalism of associating a power series to any one of the elementary functions. Candidates should be able to recognize the sine, cosine, and exponential series.

History of Mathematics

Many important developments in mathematics are too advanced to be discussed in an introductory course on the history of mathematics, yet four major developments that directly impact middle and high school mathematics deserve special attention (SMR 6.1b). The first development is the history of numeral systems through the early civilizations of Babylon, Rome, and China, and through the so-called Hindu-Arabic decimal system. A second development is the evolution of symbolic algebra, which includes contributions from Diophantus, the Hindus, Viete, and the finishing touches of Descartes. An understanding of this long and uneasy development enhances one's understanding of middle and high school mathematics as a whole. The third development is of calculus, which is rooted in ideas from Eudoxus and Archimedes, the rich but informal development of Newton and Leibniz, and the rigorous formulation that culminated with Cauchy. The fourth and last development is the concept of a proof and, therewith, the concept of an axiomatic system. Proofs formally originated with Euclid's work, and until the twentieth century, were essentially the defining characteristic of European mathematics. For almost two centuries, the questionable foundation of calculus almost forced an abandonment of the classical ideal of proofs in mathematics. It was only toward the end of the nineteenth century when proofs would again occupy center stage and a clear definition of a proof was achieved.

Subject Matter Requirements for Prospective Teachers of Mathematics³

Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

Domain 1. Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of algebra and its underlying structures, candidates have a deep conceptual knowledge. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

1.1 Algebraic Structures

- a. Know why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)
- b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
- c. Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 1.0, 2.0; Grade 7, Algebra and Functions: 1.0; Algebra I: 1.0, 3.0-7.0, 9.0-15.0, 24.0, 25.0; Geometry: 1.0, 17.0; Algebra II: 1.0-8.0, 11.0, 24.0, 25.0; Trigonometry: 17.0; Mathematical Analysis: 2.0; Linear Algebra: 9.0, 11.0)

1.2 Polynomial Equations and Inequalities

- a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming)
- b. Prove and use the following:
 - ◆ The Rational Root Theorem for polynomials with integer coefficients
 - ◆ The Factor Theorem
 - ◆ The Conjugate Roots Theorem for polynomial equations with real coefficients
 - ◆ The Quadratic Formula for real and complex quadratic polynomials
 - ◆ The Binomial Theorem
- c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-11.0, 16.0, 17.0; Trigonometry: 17.0, 18.0; Mathematical Analysis: 4.0, 6.0)

³ The Subject Matter Requirements are complemented by the **Attachment to Standard 11**, starting on page 32.

1.3 Functions

- a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
- b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
- c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 4.0-8.0, 19.0; Mathematical Analysis: 6.0, 7.0; Calculus: 9.0)

1.4 Linear Algebra

- a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products
- b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)
- c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)

(Mathematics Content Standards for California Public Schools, Algebra I: 9.0; Algebra II: 2.0; Mathematical Analysis: 1.0; Linear Algebra: 1.0-12.0)

Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

2.1 Parallelism

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
- b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

2.2 Plane Euclidean Geometry

- a. Prove theorems and solve problems involving similarity and congruence

- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)
- c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
- d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
- e. Use techniques in coordinate geometry to prove geometric theorems

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0-14.0, 18.0, 19.0; Mathematical Analysis: 5.0)

2.3 Three-Dimensional Geometry

- a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

2.4 Transformational Geometry

- a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection)
- b. Understand and prove the basic properties of dilations (e.g., similarity transformations or change of scale)

(Mathematics Content Standards for California Public Schools, Geometry: 11.0, 22.0)

Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates have a deep conceptual knowledge. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

- a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
- b. Use the Principle of Mathematical Induction to prove results in number theory
- c. Know and apply the Euclidean Algorithm

- d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 2.0; Grade 7, Number Sense: 1.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 21.0, 23.0, 25.0; Mathematical Analysis: 3.0)

Domain 4. Probability and Statistics

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of probability and statistics and their underlying structures, candidates have a deep conceptual knowledge. They solve problems and make inferences using statistics and probability distributions.

4.1 Probability

- a. Prove and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability
- d. Interpret the probability of an outcome
- e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0; Advanced Probability and Statistics: 1.0-4.0, 7.0, 9.0, 17.0, 18.0)

4.2 Statistics

- a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions
- b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions
- c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
- d. Know the method of least squares and apply it to linear regression and correlation
- e. Know and apply the chi-square test

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Advanced Probability and Statistics: 4.0-6.0, 8.0, 10.0-13.0, 15.0-17.0, 19.0)

Domain 5. Calculus*

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

5.1 Trigonometry

- Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2 x + \cos^2 x = 1$ and that this identity leads to $1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \csc^2 x$
- Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)

* *Domain 5, Calculus, does not apply to requirements for the Foundational-level Credential.*

- Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
- Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
- Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0, 14.0, 18.0, 19.0; Algebra II: 24.0, 25.0; Trigonometry: 1.0-6.0, 8.0-11.0, 19.0; Mathematical Analysis: 1.0, 2.0; Calculus: 18.0, 20.0)

5.2 Limits and Continuity

- Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
- Show that a polynomial function is continuous at a point
- Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0; Algebra II: 1.0, 15.0; Mathematical Analysis: 8.0; Calculus: 1.0-4.0)

5.3 Derivatives and Applications

- Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative
- Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
- Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
- Use the derivative to solve rectilinear motion, related rate, and optimization problems
- Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)

- f. Solve separable first-order differential equations and apply them to growth and decay problems

(Mathematics Content Standards for California Public Schools, Algebra I: 5.0-8.0, 10.0, 11.0, 13.0, 21.0, 23.0; Geometry: 3.0; Algebra II: 1.0, 9.0, 10.0, 12.0, 15.0; Trigonometry: 7.0, 15.0-19.0; Mathematical Analysis: 5.0, 7.0; Calculus: 1.0, 4.0-12.0, 27.0)

5.4 Integrals and Applications

- a. Derive definite integrals of standard algebraic functions using the formal definition of integral
- b. Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
- c. Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives
- d. Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 9.0; Calculus: 13.0-23.0)

5.5 Sequences and Series

- a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
- b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests)
- c. Calculate Taylor series and Taylor polynomials of basic functions

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0, 25.0; Algebra II: 21.0-23.0; Mathematical Analysis: 8.0; Calculus: 23.0-26.0)

Domain 6. History of Mathematics*

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries form a historical context for the content contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; e.g., numeration systems, algebra, geometry, calculus).

6.1 Chronological and Topical Development of Mathematics

- a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
- b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender

*Domain 6, History of Mathematics, does not apply to requirements for the Foundational-level Credential.

Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics

(All elements of Part II apply to both the Single Subject Credential in Mathematics and the Single Subject Credential in Foundational Mathematics.)

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they use counterexamples, construct proofs using contradictions, and create multiple representations of the same concept. They know the interconnections among mathematical ideas, and use techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing, through the use of precise language and symbols.

Candidates solve routine and complex problems by drawing from a variety of strategies while demonstrating an attitude of persistence and reflection in their approaches. They analyze problems through pattern recognition and the use of analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They make mathematical models to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(Mathematics Content Standards for California Public Schools, Grade 6, Mathematical Reasoning: 1.0-3.0; Grade 7, Mathematical Reasoning: 1.0-3.0)

Part 3: Implementation of Program Quality Standards for Subject Matter Preparation

The 2003 Program Quality Standards for Subject Matter Preparation in Mathematics are part of a broad shift in the policies of the Commission on Teacher Credentialing related to the preparation of professional teachers and other educators in California colleges and universities resulting from the mandate of Senate Bill 2042. The Commission initiated this policy change to insure high quality in educator preparation and to combine flexibility with accountability for institutions that offer programs for prospective teachers. The success of this reform effort depends on the effective *implementation* of program quality standards for each credential.

Program Equivalency

The Ryan Act established two alternatives for prospective teachers to meet the subject matter requirement:

- individuals who complete an approved subject matter program are not required to pass the subject matter examination, and
- individuals who achieve a passing score on an adopted examination are not required to complete a subject matter program.

Subject matter programs are completed by more than half of the candidates for Single Subject Credentials.

Senate Bill 2042 required that subject matter programs and examinations be aligned with the K-12 student standards and made equivalent to each other. This has been achieved in the new standards, and references are included. A candidate who completes an approved subject matter program is issued an “equivalency” to the subject matter examination.

Review and Improvement of Subject Matter Standards

The Commission will adhere to its cycle of review and reconsideration of the *Standards of Quality and Effectiveness for Subject Matter Programs in Mathematics* and in other subjects. The standards will be reviewed and reconsidered in relation to changes in academic disciplines, school curricula, and the backgrounds and needs of California students (K-12). Reviews of program standards will be based on the advice of subject matter teachers, professors and curriculum specialists. Prior to each review, the Commission will invite interested individuals and organizations to participate in the review.

Adoption and Implementation of Standards by the Commission

Program sponsors have approximately two years to transition from current to new standards of quality and effectiveness for Single Subject Matter Programs. Each sponsor is being asked to select from among seven submission deadlines during the period October 2003 through March 2005. The

form for requesting a submission date is included in this section. In the absence of a timely request for a submission date, the review may take longer. All program documents will be reviewed by statewide teams of peer reviewers selected from among qualified K–12 and IHE professional educators. It should be noted that each program of Single Subject Matter Preparation for the Single Subject Credentials must be submitted for review by the statewide panel. No new programs written to the old standards will be reviewed after the adoption of the new standards in January 2003.

Information about transition timelines for candidates, sunset dates for currently approved programs, and preconditions will be provided by the Commission through Coded Correspondence and additional program transition documents as it becomes available. Program sponsors should check the Commission website (www.ctc.ca.gov) frequently for updates.

Technical Assistance Meetings for Colleges and Universities

During April and May 2003, the Commission sponsored eight meetings to provide assistance to institutions related to their subject matter programs in mathematics. The agenda for each workshop included:

- Explanation of the implementation plan adopted by the Commission.
- Description of the steps in program review and approval.
- Review of program standards, preconditions and examples presented by Subject Matter Advisory Panel members and others with experience in implementing Standards of Program Quality.
- Opportunities to discuss subject-specific questions in small groups.

Information disseminated at those meetings is available upon request to those who were unable to attend.

Implementation Timeline: Impact on Candidates for Mathematics Credentials

Based on the Commission's implementation plan, candidates for Single Subject Credentials in Mathematics who do not plan to pass the subject matter examinations adopted by the Commission should enroll in subject matter programs that fulfill the “new” standards either (1) once a new program commences at their institution, or (2) before July 1, 2005, whichever occurs first. After a new program begins at an institution, no students should enroll for the first time in an “old” program (i.e. one approved under “old” standards). Regardless of the date when new programs are implemented, no students should enter old programs after July 1, 2005.

Candidates who enrolled in programs approved on the basis of 1994 standards (“old” programs) may complete those programs provided that (1) they entered the old programs either before new programs were available at their institutions, or before July 1, 2005, and (2) they complete the old programs before July 1, 2009. Candidates who do not comply with these timelines may qualify for Single Subject Teaching Credentials by passing the subject matter examinations that have been adopted for that purpose by the Commission.

Implementation Plan Adopted by the Commission
July 1, 2003

- (1) By July 1, 2005, existing (“old”) programs based on current guidelines should be superseded by new programs with full approval.
- (2) Once a new program receives full approval, all students not previously enrolled in the old program (i.e., all “new” students) should enroll in the new program.
- (3) After July 1, 2005, no “new” students should enroll in an “old” program, even if a new program in the subject is not available at that institution.
- (4) Students who enrolled in an old program prior to July 1, 2005, may continue to complete the old program until July 1, 2009.

Timeline for Implementing the Mathematics Standards

- January 2003** The Commission on Teacher Credentialing adopts the Standards of Program Quality and Effectiveness that are in this handbook. The Commission adopts the implementation plan outlined in this handbook. No new subject matter programs in mathematics will be reviewed in relation to the Commission's "old" standards.
- April to May 2003** The Commission conducts statewide technical assistance meetings for developing new subject matter programs to meet the new standards.
- July/October 2003** The Commission disseminates the handbook. The Commission selects, orients and trains a Program Review Panel in Mathematics. Qualified subject matter experts are prepared to review programs in relation to the standards beginning in 2003-04.
- October 2003** Review and approval of programs under the new standards begins.
- 2003-05** Institutions may submit programs for review on or after October 1, 2003, after requesting and being assigned a submission date by Commission staff. Once a "new" program is approved, all students who were not previously enrolled in the "old" program (i.e., all new students) should enroll in the new program. Students may complete an old program if they enrolled in it either (1) prior to the commencement of the new program at their campus, or (2) prior to July 1, 2005, whichever occurs first.
- July 1, 2005** "Old" programs that are based on 1994 standards must be superseded by new programs with full approval (see pages 42-43). After July 1, 2005, no new students may enroll in an old program, even if a new program in mathematics is not yet available at the institution.
- 2005-09** The Commission will continue to review program proposals based on the standards and preconditions in this handbook. Institutions which submit program proposals without an assigned submission date will be reviewed at the earliest date of an opening in the submission schedule.
- July 1, 2009** The final date for candidates to complete subject matter preparation programs approved under the 1994 standards. To qualify for a credential based on an "old" program, students must have entered that program prior to either (1) the implementation of a new program with full or interim approval at their institution, or (2) July 1 2005, whichever occurs first.

Implementation Timeline Diagram

January 2003

Adopt the mathematics standards and preconditions in this handbook, including the implementation plan.

January to May, 2003

Disseminate the standards, timeline and implementation plan throughout the state. Hold regional technical assistance meetings to offer information, answer questions, and assist colleges and universities in developing new programs.

October 2003

Colleges and universities may begin to present program documents for review by the Commission's staff and Program Review Panels.

July 1, 2005

"Old" subject matter programs in mathematics must be superseded by new approved programs.

July 1, 2009

Final date for candidates to qualify for Single Subject Credentials in Mathematics on the basis of "old" programs of subject matter preparation.

Review and Approval of Mathematics Subject Matter Programs

A regionally accredited institution of post-secondary education that would like to offer (or continue to offer) a Program of Subject Matter Preparation for the Single Subject Credential in Mathematics may present a program proposal that responds to the standards and preconditions in this handbook. The submission of programs for review and approval is voluntary for colleges and universities.

If an institution would like to offer two or more distinct programs of subject matter preparation in mathematics, a separate proposal may be forwarded to the Commission for each program. For example, one program in mathematics might emphasize studies of mathematical applications, while a second program at the same institution could have an emphasis in computer science. However, the Commission encourages institutions to coordinate its single subject programs that are within the same subject matter discipline.

The Commission is prepared to review subject matter program proposals beginning on October 1, 2003. Prior to that date, the Commission's professional staff is able to consult with institutional representatives on meeting the new standards and preparing program documents.

Selection, Composition and Training of Program Review Panels

Review panel members are selected because of their expertise in mathematics, and their knowledge of mathematics curriculum and instruction in the public schools of California. Reviewers are selected from institutions of higher education, school districts, county offices of education, organizations of subject matter experts, and statewide professional organizations. Members are selected according to the Commission's adopted policies that govern the selection of panels. Members of the Commission's former Single Subject Waiver Panels and Subject Matter Advisory Panels may be selected to serve on Program Review Panels.

The Commission staff conducts a training and calibration session that all reviewers must attend. Training includes:

- The purpose and function of subject matter preparation programs.
- The Commission's legal responsibilities in program review and approval.
- The role of the review panel in making program determinations.
- The role of the Commission's professional staff in assisting the panel.
- A thorough analysis and discussion of each standard and rationale.
- Alternative ways in which the standard could be met.
- An overview of review panel procedures.
- Simulated practice and calibration in reviewing programs.
- Responsive feedback for program revision.

Steps in the Review of Programs

The Commission is committed to conducting a program review process that is objective, authoritative and comprehensive. The agency also seeks to be as helpful as possible to colleges and

universities throughout the review process. Commission staff is available to consult with during program document development.

Review of Preconditions. An institution's response to the preconditions is reviewed by the Commission's professional staff because the preconditions are based on Commission policies and do not involve issues of program quality. Preconditions are reviewed upon the institution's formal submission of a document. Once the status of the preconditions is established, the program document is referred to the expert review panel.

Review of Program Quality Standards. Unlike the preconditions, the standards address issues of program quality and effectiveness, so each institution's response to the standards is reviewed by a small Program Review Panel of subject matter experts. If the Program Review Panel determines that a proposed program fulfills the standards, the Commission's staff recommends the program for approval by the Commission during a public meeting no more than eight weeks after the panel's decision.

If the Program Review Panel determines that the program does not meet the standards, the document is returned to the institution with an explanation of the panel's findings. Specific reasons for the panel's decision are communicated to the institution. If the panel has substantive concerns about one or more aspects of program quality, representatives of the institution can obtain information and assistance from the Commission's staff.

The Commission would like the program review process to be as helpful as possible to colleges and universities. Because a large number of institutions prepare teachers in California, representatives of an institution should first consult with the Commission's professional staff regarding programs that are in preparation or under review. The staff responds to all inquiries expeditiously and knowledgeably. Representatives of colleges and universities should contact members of a Program Review Panel only when they are authorized to do so by the Commission's staff. This restriction must be observed to ensure that membership on a panel is manageable for the reviewers. If an institution finds that needed information is not sufficiently available, please inform the designated staff consultant. If the problem is not corrected in a timely way, please contact the executive director of the Commission. After changes have been made in the program, the proposal may be re-submitted to the Commission's staff for reconsideration by the panel.

If the Program Review Panel determines that minor or technical changes should be made in a program, the responsibility for reviewing the resubmitted document rests with the Commission's professional staff, which presents the *revised* program to the Commission for approval without further review by the panel.

Appeal of an Adverse Decision. An institution that would like to appeal a decision of the staff (regarding preconditions) or the Program Review Panel (regarding standards) may do so by submitting the appeal to the executive director of the Commission. The institution should include the following information in the appeal:

- The original program document and the stated reasons of the Commission's staff or the review panel for not recommending approval of the program.
- A specific response by the institution to the initial denial, including a copy of the resubmitted document (if it has been resubmitted).

- A rationale for the appeal by the institution.

The executive director may deny the appeal, or appoint an independent review panel, or present the appeal directly to the Commission for consideration.

Submission Guidelines for Single Subject Matter Program Documents

To facilitate the proposal review and approval process, Commission staff has developed the following instructions for organizations submitting documents for approval of Single Subject Matter Programs. It is essential that these instructions be followed accurately. Failure to comply with these procedures can result in a proposal being returned to the prospective program sponsor for reformatting and/or revision prior to being forwarded to program reviewers.

Transmittal Instructions

Sponsoring agencies are required to submit **one printed bound paper copy** of their proposal(s), to the following address:

California Commission on Teacher Credentialing
Professional Services Division: Single Subject Matter Programs
1900 Capitol Avenue
Sacramento, CA 95814

In addition, **one electronic copy of the proposal text** (including supporting evidence where possible) should be submitted in Microsoft Word, or a Microsoft Word compatible format. Some phases of the review process will involve secure web-based editing. To facilitate this process, please leave no spaces in the name of your document, and be sure that the name of the file ends in ".doc" (example: CTCdocument.doc).

Submittal Deadlines

There are seven opportunities during which to submit proposals for review and approval. The submittal deadlines are:

October 1, 2003	August 2, 2004
January 5, 2004	November 2, 2004
March 2, 2004	March 1, 2005*
June 1, 2004	

*Any programs submitted after 2005 will be reviewed according to the availability of the review panel.

Organization of Required Documents

Sponsoring agencies should include as the cover page of each copy of the program application the “Sponsoring Organization Transmittal Cover Sheet.” A copy of the Transmittal Cover Sheet is located at the end of this section of the handbook for use by program sponsors. The proposal application documents should begin with Transmittal Cover Sheet that includes the original signatures of the program contacts and chief executive officer.

The program contact identified on the Transmittal Cover Sheet will be the individual who is informed electronically and by mail as changes occur, and to whom the review feedback will be sent. Program sponsors are strongly urged to consult the CTC web site, www.ctc.ca.gov , for updates relating to the implementation of new single subject matter standards and programs.

Each proposal must be organized in the following order:

- Transmittal Cover Sheet
- Table of Contents
- Responses to Preconditions, including course lists, units and catalog descriptions
- A matrix identifying which courses meet which subject matter requirements
- One to two pages of narrative response to each Standard

The response to the standards must:

- be tabbed/labeled to help guide the reviewers,
- have numbered pages,
- provide supporting evidence included after each response or organized into appendices.
- evidence should be cross-referenced or electronically linked in the response, and appendices *must* be tabbed and labelled for easy access by reviewers.

Responding to the Standards Common To All

The Commission adopted two standards that relate to program design and structure for programs in *all* single subject disciplines:

Standard 1	Program Design
Standard 2	Program Resources and Support

These two standards are referred to as “standards common to all” because they are the same in all subject areas. An institution’s program document should include a subject-specific reply to both standards.

Responses to the Program Standards

Program proposals should provide sufficient information about how the program intends to deliver content consistent with each standard so that a knowledgeable team of professionals can determine whether each standard has been met by the program. The goal in writing the response to any standard should be to describe the proposed program clearly enough for an outside reader to understand what a prospective teacher will experience, as he or she progresses through the program in terms of depth, breadth, and sequencing of instructional and field experiences, and what he or she will know and be able to do and demonstrate at the end of the program. Review teams will then be able to assess the responses for consistency with the standard, completeness of the response, and quality of the supporting evidence.

The written text should be organized in the same order as the standards. Responses should not merely reiterate the standard. They should describe how the standard will be met in the coursework content, requirements, and processes and by providing evidence from course syllabi or other course materials to support the explanation. ***Responses that do not completely address each standard will be considered incomplete and returned for revision.***

Lines of suitable evidence will vary with each standard. Some examples of evidence helpful for review teams include:

- Charts and graphic organizers to illustrate program organization and design
- Course or module outlines, or showing the sequence of course topics, classroom activities, materials and texts used, and out-of-class assignments
- Specific descriptions of assignments and other formative assessments that demonstrate how prospective teachers will reinforce and extend key concepts and/or demonstrate an ability or competence
- Documentation of materials to be used, including tables of contents of textbooks and identification of assignments from the texts, and citations for other reading assignments.
- Current catalog descriptions.

Packaging A Submission for Shipment to the Commission

Please do **not**:

- Use foam peanuts as packaging material
- Overstuff the binders. Use two binders if necessary.
- Overstuff the boxes in which the binders are packed, as these may break open in shipment.

**Submission Request Form
For Single Subject Matter Preparation Program Response to Standards**

Program Sponsor (Name of Institution and Department)

Please fill out the requested information below to help us plan for providing technical assistance in a timely manner.

Contact Person: _____ **Title:** _____

Department: _____

Address: _____

Phone: _____ **Fax:** _____

Email: _____

Please indicate the subject area for which you are submitting a program proposal document:

English _____ Mathematics _____ Science _____ Social Science _____

Please indicate when you intend to submit program documents responding to the new Single Subject Matter Preparation Standards: _____

Submit to: Commission on Teacher Credentialing
Professional Services Division:
Single Subject Matter Programs
1900 Capitol Ave.
Sacramento, CA 95814
Fax (916) 324-8927

Single Subject Matter Program Sponsor - Transmittal Cover Sheet
(Page 1 of 2)

• **Date** _____

• **Sponsoring Organization:**

Name _____

• **Submission Type(s) Place a check mark in the appropriate box.**

English Subject Matter Preparation	
Mathematics Subject Matter Preparation	
Science Subject Matter Preparation	
Social Science Subject Matter Preparation	

• **Program Contacts:**

1. Name _____

Title _____

Address _____

Phone _____ Fax _____

E-mail _____

**Single Subject Program Sponsor - Transmittal Cover Sheet
(Page 2 of 2)**

Name _____

Title _____

Address _____

Phone _____ Fax _____

E-mail _____

Chief Executive Officer (*President or Provost; Superintendent*):

Name _____

Address _____

Phone _____ Fax _____

E-mail _____

I Hereby Signify My Approval to Transmit This Program Document to the California Commission on Teacher Credentialing:

CEO Signature _____

Title _____

Date _____

Appendix A
Assembly Bill No. 537
(Education Code Chapter 587, Statutes of 1999)

CHAPTER 587

An act to amend Sections 200, 220, 66251, and 66270 of, to add Section 241 to, and to amend and renumber Sections 221 and 66271 of, the Education Code, relating to discrimination.

[Approved by Governor October 2, 1999. Filed with Secretary of State October 10, 1999.]

LEGISLATIVE COUNSEL'S DIGEST

AB 537, Kuehl. Discrimination.

(1) Existing law provides that it is the policy of the State of California to afford all persons in public schools and postsecondary institutions, regardless of their sex, ethnic group identification, race, national origin, religion, or mental or physical disability, equal rights and opportunities in the educational institutions of the state.

Existing law makes it a crime for a person, whether or not acting under color of law, to willfully injure, intimidate, interfere with, oppress, or threaten any other person, by force or threat of force, in the free exercise or enjoyment of any right or privilege secured to him or her by the Constitution or laws of this state or by the Constitution or laws of the United States because of the other person's race, color, religion, ancestry, national origin, disability, gender, or sexual orientation, or because he or she perceives that the other person has one or more of those characteristics.

This bill would also provide that it is the policy of the state to afford all persons in public school and postsecondary institutions equal rights and opportunities in the educational institutions of the state, regardless of any basis referred to in the aforementioned paragraph.

(2) Existing law prohibits a person from being subjected to discrimination on the basis of sex, ethnic group identification, race, national origin, religion, color, or mental or physical disability in any program or activity conducted by any educational institution or postsecondary educational institution that receives, or benefits from, state financial assistance or enrolls students who receive state student financial aid.

This bill would also prohibit a person from being subjected to discrimination on the basis of any basis referred to in paragraph (1) in any program or activity conducted by any educational institution or postsecondary educational institution that receives, or benefits from, state financial assistance or enrolls students who receive state student financial aid.

(3) This bill would state that it does not require the inclusion of any curriculum, textbook, presentation, or other material in any program or activity conducted by an educational institution or a postsecondary educational institution and would prohibit this bill from being deemed to be violated by the omission of any curriculum, textbook, presentation, or other material in any program or activity conducted by an educational institution or a postsecondary educational institution.

To the extent that this bill would impose new duties on school districts and community college districts, it would impose a state-mandated local program.

(4) The California Constitution requires the state to reimburse local agencies and school districts for certain costs mandated by the state. Statutory provisions establish procedures for making that reimbursement, including the creation of a State Mandates Claims Fund to pay the costs of mandates that do not exceed \$1,000,000 statewide and other procedures for claims whose statewide costs exceed \$1,000,000.

This bill would provide that, if the Commission on State Mandates determines that the bill contains costs mandated by the state, reimbursement for those costs shall be made pursuant to these statutory provisions.

The people of the State of California do enact as follows:

SECTION 1. This bill shall be known, and may be cited, as the California Student Safety and Violence Prevention Act of 2000.

SEC. 2. (a) The Legislature finds and declares all of the following:

(1) Under the California Constitution, all students of public schools have the inalienable right to attend campuses that are safe, secure, and peaceful. Violence is the number one cause of death for young people in California and has become a public health problem of epidemic proportion. One of the Legislature's highest priorities must be to prevent our children from the plague of violence.

(2) The fastest growing, violent crime in California is hate crime, and it is incumbent upon us to ensure that all students attending public school in California are protected from potentially violent discrimination. Educators see how violence affects youth every day; they know first hand that youth cannot learn if they are concerned about their safety. This legislation is designed to protect the institution of learning as well as our students.

(3) Not only do we need to address the issue of school violence but also we must strive to reverse the increase in teen suicide. The number of teens who attempt suicide, as well as the number who actually kill themselves, has risen substantially in recent years. Teen suicides in the United States have doubled in number since 1960 and every year over a quarter of a million adolescents in the United States attempt suicide. Sadly, approximately 4,000 of these attempts every year are completed. Suicide is the third leading cause of death for youths 15 through 24 years of age. To combat this problem we must seriously examine these grim statistics and take immediate action to ensure all students are offered equal protection from discrimination under California law.

SEC. 3. Section 200 of the Education Code is amended to read:

200. It is the policy of the State of California to afford all persons in public schools, regardless of their sex, ethnic group identification, race, national origin, religion, mental or physical disability, or regardless of any basis that is contained in the prohibition of hate crimes set forth in subdivision (a) of Section 422.6 of the Penal Code, equal rights and opportunities in the educational institutions of the state. The purpose of this chapter is to prohibit acts which are contrary to that policy and to provide remedies therefor.

SEC. 4. Section 220 of the Education Code is amended to read:

220. No person shall be subjected to discrimination on the basis of sex, ethnic group identification, race, national origin, religion, color, mental or physical disability, or any basis that is contained in the prohibition of hate crimes set forth in subdivision (a) of Section 422.6 of the Penal Code in any program or activity conducted by an educational institution that receives, or benefits from, state financial assistance or enrolls pupils who receive state student financial aid.

SEC. 5. Section 221 of the Education Code is renumbered to read:

220.5. This article shall not apply to an educational institution which is controlled by a religious organization if the application would not be consistent with the religious tenets of that organization.

SEC. 6. Section 241 is added to the Education Code, to read:

241. Nothing in the California Student Safety and Violence Prevention Act of 2000 requires the inclusion of any curriculum, textbook, presentation, or other material in any program or activity conducted by an educational institution or postsecondary educational institution; the California Student Safety and Violence Prevention Act of 2000 shall not be deemed to be violated by the omission of any curriculum, textbook, presentation, or other material in any program or activity conducted by an educational institution or postsecondary educational institution.

SEC. 7. Section 66251 of the Education Code is amended to read:

66251. It is the policy of the State of California to afford all persons, regardless of their sex, ethnic group identification, race, national origin, religion, mental or physical disability, or regardless of any basis that is contained in the prohibition of hate crimes set forth in subdivision (a) of Section 422.6 of the Penal Code, equal rights and opportunities in the postsecondary institutions of the state. The purpose of this chapter is to prohibit acts that are contrary to that policy and to provide remedies therefor.

SEC. 8. Section 66270 of the Education Code is amended to read:

66270. No person shall be subjected to discrimination on the basis of sex, ethnic group identification, race, national origin, religion, color, or mental or physical disability, or any basis that is contained in the prohibition of hate crimes set forth in subdivision (a) of Section 422.6 of the Penal Code in any program or activity conducted by any postsecondary educational institution that receives, or benefits from, state financial assistance or enrolls students who receive state student financial aid.

SEC. 9. Section 66271 of the Education Code is renumbered to read:

66270.5. This chapter shall not apply to an educational institution that is controlled by a religious organization if the application would not be consistent with the religious tenets of that organization.

SEC. 10. Notwithstanding Section 17610 of the Government Code, if the Commission on State Mandates determines that this act contains costs mandated by the state, reimbursement to local agencies and school districts for those costs shall be made pursuant to Part 7 (commencing with Section 17500) of Division 4 of Title 2 of the Government Code. If the statewide cost of the claim for reimbursement does not exceed one million dollars (\$1,000,000), reimbursement shall be made from the State Mandates Claims Fund.

