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Action

Professional Services Committee

Approval of Revised Preconditions for Foundational-Level Mathematics Subject Matter Programs

Executive Summary: This agenda item presents a recommendation for revised Preconditions for Foundational-Level Mathematics Subject Matter Programs.

Recommended Action: Staff recommends adoption of the proposed revised Preconditions for the Foundational-Level Mathematics subject matter preparation programs.

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Strategic Plan Goal: 1

Promote educational excellence through the preparation and certification of professional educators

- ◆ Sustain high quality standards for the preparation and performance of professional educators and for the accreditation of credential programs

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Approval of Revised Preconditions for Foundational-Level Mathematics Subject Matter Programs

Introduction

Preconditions for Foundational-Level Mathematics Subject Matter Programs (FLM) were adopted in 2003 and require the same number of units (45) as the preconditions for full mathematics subject matter programs. However, no FLM subject matter programs have been approved because the requirement of 45 semester units is not feasible for the scope of content ascribed to the foundational-level mathematics program.

In January 2011 the Commission adopted preconditions for the Foundational-Level General Science (FLGS) Subject Matter Programs, including the requirement of 32 semester units (<http://www.ctc.ca.gov/commission/agendas/2011-01/2011-01-2C.pdf>). This agenda item presents a recommendation for revised Preconditions for Foundational-Level Mathematics consistent with the recently adopted preconditions for FLGS programs.

Background

For most of the Commission's teaching credentials, an individual may satisfy the subject matter requirement by passing the appropriate Commission-approved subject matter examination, currently the California Subject Examinations for Teachers (CSET), or by completing a Commission-approved subject matter preparation program. Due to California's approved federal No Child Left Behind (NCLB) compliance plan, all prospective multiple subject teachers must take and pass the CSET-Multiple Subjects examination.

A Commission-approved subject matter preparation program is defined by the appropriate adopted program standards and preconditions (<http://www.ctc.ca.gov/educator-prep/STDS-subject-matter.html>). The specific content (depth and breadth) of the subject matter preparation program is defined by the subject matter requirements (SMRs—Appendix C), and the adopted standards address issues of quality of the preparation program. Preconditions define the number of units required in a program and the content areas within the subject which must be included in the program. Programs must meet these requirements as well as the program standards for approval by the Commission. Provided below is the definition from the Single Subject Matter Handbooks regarding Preconditions:

A precondition is a requirement for initial and continued program approval. Unlike standards, preconditions specify requirements for program compliance, not program quality. The Commission determines whether a program complies with the adopted preconditions on the basis of a program document provided by the college or university. In the program review sequence, a program that meets all preconditions is eligible for a more intensive review to determine if the program's quality satisfies the Commission's standards.

The Preconditions for Subject Matter Programs in Foundational-Level Mathematics (Appendix A) currently require 45 units of coursework, the same number as the full single subject mathematics program. The primary difference between the two programs is that the Foundational-Level Mathematics Program may also include 15 units of mathematics-based courses, such as engineering, physics and computer science, rather than the extended study of mathematics required for the Mathematics subject matter program. However, 30 units of mathematics courses are required for both programs. The courses appropriate for the Foundational-Level Mathematics program are often lower division mathematics courses. Programs may also choose to include coursework at higher levels at their discretion. The Preconditions for the full Mathematics subject matter programs are presented as Appendix B.

Authorization for the Foundational-Level Mathematics Subject Matter Credential

The Single Subject Teaching Credential in Foundational-Level Mathematics authorizes the holder to provide instruction in the content taught to the majority of California's K-12 public school math students: general mathematics, algebra, geometry, probability and statistics, and consumer mathematics. Instruction is permitted in grades twelve and below, including preschool and classes organized primarily for adults. However, this credential does not authorize teaching of higher levels of mathematics commonly taught in the higher secondary grades and beyond the scope of the mathematics content areas noted above. The Subject Matter Requirements for Mathematics subject matter programs are provided in Appendix C. The full mathematics subject matter programs must address all six domains while the Foundational-Level Mathematics programs only address the first four domains.

Rationale for the Revised FLM Preconditions

At the time that the Commission established the Foundational-Level Mathematics Single Subject Credential, it did so to provide additional pathways to teaching in a subject area that was, and continues to be, in critical need of qualified teachers. If the Commission intends to continue to provide two routes for individuals to demonstrate subject matter competency for the Foundational-Level Mathematics subject matter requirement (i.e., a program route and an examination route), the number of required units for the program option may need to be adjusted to fit the scope of content required for the credential. An individual must pass all three of the CSET Mathematics subtests to meet the full Mathematics subject matter requirement, while only two subtests are required for the Foundational-Level Mathematics authorization. In summary, staff proposes revising the FLM Preconditions for the following reasons:

1. The Commission's commitment to offer multiple routes to meet subject matter requirements
2. The lack of sufficient lower level mathematics coursework available at the postsecondary level
3. The narrower scope of subject matter content required for FLM
4. The NCLB legislation requires 32 semester units as a measure of a highly qualified teacher
5. The shortage of fully- prepared mathematics teachers

The following table shows the recommended revision of the Commission's current requirements pertaining to Mathematics credentials.

Required Semester Units	Adopted		Proposed Foundational Mathematics
	Mathematics	Foundational Mathematics	
Total	45	45	32
Core Mathematics Courses	30	30	20
Extended Study of Mathematics Courses	15		12
Extended Study of Mathematics-based or Affiliated Courses		15	

Next Steps

Staff recommends adoption of the proposed revised Preconditions for the Foundational-Level Mathematics subject matter preparation programs. If the Commission adopts the revised Preconditions, staff would immediately issue a Program Sponsor Alert and begin working with program sponsors to develop programs to submit for review and approval.

Program sponsors would be allowed to apply for approval of Foundational-Level Mathematics subject-matter programs. Programs that are currently approved to offer the full mathematics subject-matter programs would be allowed to recommend candidates for the FLM credential immediately through written statement to the Commission that the program intends to use its approved general mathematics coursework as an approved FLM subject-matter program.

Appendix A

Adopted Preconditions for the Approval of Subject Matter Programs in Foundational-Level Mathematics

To be approved by the Commission, a Subject Matter Program in Foundational-Level Mathematics must comply with the following preconditions.

- (1) Each program of subject matter preparation for the Single Subject Teaching Credential in Foundational-Level Mathematics shall include (a) a minimum of 30 semester units (or 45 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides extended study of the subject. These two requirements are elaborated in Preconditions 2 and 3.
- (2) The core of the program shall include coursework in subjects commonly taught in departmentalized classes of mathematics and related subjects in the California public schools such as algebra (or demonstrated proficiency), geometry, number theory, and statistics and probability.
- (3) Extended studies (breadth, depth, perspective, concentrations) in the program shall be designed to supplement the core of the program.

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include the course titles, unit designations, catalog descriptions and syllabi of all courses in the program that are used to meet the standards. Program documents must also include a matrix that identifies which courses meet which subject matter requirements.

Institutions may determine whether the standards are addressed through one or more courses for each commonly taught subject or courses offering integrated study of these subjects. Institutions may also define the program in terms of required or elective coursework. However, elective options must all meet the standards. Coursework offered by any appropriate department(s) of a regionally accredited institution may satisfy the preconditions and standards in this handbook. Programs may use general education courses in meeting the standards.

Appendix B

Adopted Preconditions for the Approval of Subject Matter Programs in Mathematics

To be approved by the Commission, a Subject Matter Program in Mathematics must comply with the following preconditions.

- (1) Each program of subject matter preparation for the Single Subject Teaching Credential in Mathematics shall include (a) a minimum of 30 semester units (or 45 quarter units) of core mathematics coursework that is directly related to subjects that are commonly taught in departmentalized mathematics classes in California public schools, and (b) a minimum of 15 semester units (or 22 quarter units) of coursework that provides extended study of the subject. These two requirements are elaborated in Preconditions 2 and 3.
- (2) The core of the program shall include coursework in subjects commonly taught in departmentalized classes of mathematics and related subjects in the California public schools such as algebra (or demonstrated proficiency), geometry, number theory, calculus, history of mathematics, and statistics and probability.
- (3) Extended studies (breadth, depth, perspective, concentrations) in the program shall be designed to supplement the core of the program.

In addition to describing how a program meets each standard of program quality in this handbook, the program document by an institution shall include the course titles, unit designations, catalog descriptions and syllabi of all courses in the program that are used to meet the standards. Program documents must also include a matrix that identifies which courses meet which subject matter requirements.

Institutions may determine whether the standards are addressed through one or more courses for each commonly taught subject or courses offering integrated study of these subjects. Institutions may also define the program in terms of required or elective coursework. However, elective options must all meet the standards. Coursework offered by any appropriate department(s) of a regionally accredited institution may satisfy the preconditions and standards in this handbook. Programs may use general education courses in meeting the standards.

Appendix C

Subject Matter Requirements for Prospective Teachers of Mathematics

Part I: Content Domains for Subject Matter Understanding and Skill in Mathematics

Domain 1. Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of algebra and its underlying structures, candidates have a deep conceptual knowledge. They are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations. They understand the power of mathematical abstraction and symbolism.

1.1 Algebraic Structures

- a. Know why the real and complex numbers are each a field, and that particular rings are not fields (e.g., integers, polynomial rings, matrix rings)
- b. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
- c. Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 1.0, 2.0; Grade 7, Algebra and Functions: 1.0; Algebra I: 1.0, 3.0-7.0, 9.0-15.0, 24.0, 25.0; Geometry: 1.0, 17.0; Algebra II: 1.0-8.0, 11.0, 24.0, 25.0; Trigonometry: 17.0; Mathematical Analysis: 2.0; Linear Algebra: 9.0, 11.0)

1.2 Polynomial Equations and Inequalities

- a. Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming)
- b. Prove and use the following:
 - ◆ The Rational Root Theorem for polynomials with integer coefficients
 - ◆ The Factor Theorem
 - ◆ The Conjugate Roots Theorem for polynomial equations with real coefficients
 - ◆ The Quadratic Formula for real and complex quadratic polynomials
 - ◆ The Binomial Theorem
- c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-11.0, 16.0, 17.0; Trigonometry: 17.0, 18.0; Mathematical Analysis: 4.0, 6.0)

1.3 Functions

- a. Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
- b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)
- c. Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 4.0-8.0, 19.0; Mathematical Analysis: 6.0, 7.0; Calculus: 9.0)

1.4 Linear Algebra

- a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products
- b. Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product)
- c. Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations)

(Mathematics Content Standards for California Public Schools, Algebra I: 9.0; Algebra II: 2.0; Mathematical Analysis: 1.0; Linear Algebra: 1.0-12.0)

Domain 2. Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of geometry and its underlying structures, candidates have a deep conceptual knowledge. They demonstrate an understanding of axiomatic systems and different forms of logical arguments. Candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry, including coordinate, synthetic, non-Euclidean, and transformational geometry.

2.1 Parallelism

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)
- b. Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

2.2 Plane Euclidean Geometry

- a. Prove theorems and solve problems involving similarity and congruence
- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)
- c. Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
- d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8)
- e. Use techniques in coordinate geometry to prove geometric theorems

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0-14.0, 18.0, 19.0; Mathematical Analysis: 5.0)

2.3 Three-Dimensional Geometry

- a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

2.4 Transformational Geometry

- a. Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection)
- b. Understand and prove the basic properties of dilations (e.g., similarity transformations or change of scale)

(Mathematics Content Standards for California Public Schools, Geometry: 11.0, 22.0)

Domain 3. Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of number theory and its underlying structures, candidates have a deep conceptual knowledge. They prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

- a. Prove and use basic properties of natural numbers (e.g., properties of divisibility)
- b. Use the Principle of Mathematical Induction to prove results in number theory
- c. Know and apply the Euclidean Algorithm
- d. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 2.0; Grade 7, Number Sense: 1.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 21.0, 23.0, 25.0; Mathematical Analysis: 3.0)

Domain 4. Probability and Statistics

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of probability and statistics and their underlying structures, candidates have a deep conceptual knowledge. They solve problems and make inferences using statistics and probability distributions.

4.1 Probability

- a. Prove and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability
- d. Interpret the probability of an outcome

- e. Use normal, binomial, and exponential distributions to solve and interpret probability problems

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0; Advanced Probability and Statistics: 1.0-4.0, 7.0, 9.0, 17.0, 18.0)

4.2 Statistics

- a. Compute and interpret the mean, median, and mode of both discrete and continuous distributions
- b. Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions
- c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results
- d. Know the method of least squares and apply it to linear regression and correlation
- e. Know and apply the chi-square test

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0; Advanced Probability and Statistics: 4.0-6.0, 8.0, 10.0-13.0, 15.0-17.0, 19.0)

Domain 5. Calculus*

Candidates demonstrate an understanding of the trigonometry and calculus contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999) from an advanced standpoint. To ensure a rigorous view of trigonometry and calculus and their underlying structures, candidates have a deep conceptual knowledge. They apply the concepts of trigonometry and calculus to solving problems in real-world situations.

5.1 Trigonometry

- a. Prove that the Pythagorean Theorem is equivalent to the trigonometric identity $\sin^2x + \cos^2x = 1$ and that this identity leads to $1 + \tan^2x = \sec^2x$ and $1 + \cot^2x = \csc^2x$
- b. Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle)
- c. Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems)
- d. Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan)
- e. Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem)

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0, 14.0, 18.0, 19.0; Algebra II: 24.0, 25.0; Trigonometry: 1.0-6.0, 8.0-11.0, 19.0; Mathematical Analysis: 1.0, 2.0; Calculus: 18.0, 20.0)

5.2 Limits and Continuity

- Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit
- Show that a polynomial function is continuous at a point
- Know and apply the Intermediate Value Theorem, using the geometric implications of continuity

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 3.0; Algebra II: 1.0, 15.0; Mathematical Analysis: 8.0; Calculus: 1.0-4.0)

5.3 Derivatives and Applications

- Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative
- Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change)
- Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule
- Use the derivative to solve rectilinear motion, related rate, and optimization problems
- Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity)
- Solve separable first-order differential equations and apply them to growth and decay problems

(Mathematics Content Standards for California Public Schools, Algebra I: 5.0-8.0, 10.0, 11.0, 13.0, 21.0, 23.0; Geometry: 3.0; Algebra II: 1.0, 9.0, 10.0, 12.0, 15.0; Trigonometry: 7.0, 15.0-19.0; Mathematical Analysis: 5.0, 7.0; Calculus: 1.0, 4.0-12.0, 27.0)

5.4 Integrals and Applications

- Derive definite integrals of standard algebraic functions using the formal definition of integral
- Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums)
- Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives
- Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0; Geometry: 9.0; Calculus: 13.0-23.0)

5.5 Sequences and Series

- a. Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number)
- b. Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests)
- c. Calculate Taylor series and Taylor polynomials of basic functions

(Mathematics Content Standards for California Public Schools, Algebra I: 24.0, 25.0; Algebra II: 21.0-23.0; Mathematical Analysis: 8.0; Calculus: 23.0-26.0)

** Domain 5, Calculus, does not apply to requirements for the Foundational-level Credential.*

Domain 6. History of Mathematics*

Candidates understand the chronological and topical development of mathematics and the contributions of historical figures of various times and cultures. Candidates know important mathematical discoveries and their impact on human society and thought. These discoveries form a historical context for the content contained in the Mathematics Content Standards for California Public Schools (1997) as outlined in the Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (1999; e.g., numeration systems, algebra, geometry, calculus).

6.1 Chronological and Topical Development of Mathematics

- a. Demonstrate understanding of the development of mathematics, its cultural connections, and its contributions to society
- b. Demonstrate understanding of the historical development of mathematics, including the contributions of diverse populations as determined by race, ethnicity, culture, geography, and gender

**Domain 6, History of Mathematics, does not apply to requirements for the Foundational-level Credential.*

Part II: Subject Matter Skills and Abilities Applicable to the Content Domains in Mathematics

(All elements of Part II apply to both the Single Subject Credential in Mathematics and the Single Subject Credential in Foundational Mathematics.)

Candidates for Single Subject Teaching Credentials in mathematics use inductive and deductive reasoning to develop, analyze, draw conclusions, and validate conjectures and arguments. As they reason, they use counterexamples, construct proofs using contradictions, and create multiple representations of the same concept. They know the interconnections among mathematical ideas, and use techniques and concepts from different domains and sub-domains to model the same problem. They explain mathematical interconnections with other disciplines. They are able to communicate their mathematical thinking clearly and coherently to others, orally, graphically, and in writing, through the use of precise language and symbols.

Candidates solve routine and complex problems by drawing from a variety of strategies while demonstrating an attitude of persistence and reflection in their approaches. They analyze problems through pattern recognition and the use of analogies. They formulate and prove conjectures, and test conclusions for reasonableness and accuracy. They use counterexamples to disprove conjectures.

Candidates select and use different representational systems (e.g., coordinates, graphs). They understand the usefulness of transformations and symmetry to help analyze and simplify problems. They make mathematical models to analyze mathematical structures in real contexts. They use spatial reasoning to model and solve problems that cross disciplines.

(Mathematics Content Standards for California Public Schools, Grade 6, Mathematical Reasoning: 1.0-3.0; Grade 7, Mathematical Reasoning: 1.0-3.0)