
2B

Action

Professional Services Committee

Proposed Preconditions and Program Standards for the Teaching of Mathematics

Executive Summary: This agenda item presents, for adoption, the proposed preconditions and proposed program standards for the Mathematics Instructional Certificate and Mathematics Instructional Leadership Specialist Credential.

Recommended Action: Staff recommends Commission adoption of the proposed preconditions and proposed program standards for the Mathematics Instructional Certificate and Mathematics Instructional Leadership Specialist Credential.

Presenter: Rebecca Parker, Consultant and Terry Janicki, Administrator, Professional Services Division

Strategic Plan Goal: 1

Promote educational excellence through the preparation and certification of professional educators

- ◆ Sustain high quality standards for the preparation and performance of professional educators and for the accreditation of credential programs

September-October 2010

Proposed Preconditions and Program Standards for the Teaching of Mathematics

Introduction

This agenda item presents, for adoption, the proposed preconditions and proposed program standards for the Mathematics Instructional Certificate and Mathematics Instructional Leadership Specialist Credential. A draft of these standards was presented to the Commission in June 2010. A survey to collect comments about the draft standards from the field was distributed widely through professional mathematics teacher associations and through the PSD e-news. The panel met in June to review comments from the field. Based on the panel's review of stakeholder comments and the June Commission discussion and direction, the panel edited the draft program standards for the Mathematics Instructional Certificate (MIC) and the Mathematics Instructional Leadership (MIL) Specialist credential as appropriate.

Background

Currently a variety of authorizations allow a certificated individual to teach mathematics in California's public schools. An agenda item describing all authorizations to teach mathematics was presented to the Commission in January 2009 (<http://www.ctc.ca.gov/commission/agendas/2009-01/2009-01-3E.pdf>).

In order to look more closely at the issues raised by the January 2009 agenda item, the Commission directed staff to convene a Teaching Mathematics Advisory Panel (TMAP) which would include representatives from key stakeholder groups. Staff developed an application that was publicized widely through the Commission's website, mathematics professional associations, the California Subject Matter Projects, and the Professional Services Division weekly e-news. The twenty member panel (see Appendix A) was appointed by Executive Director Dale Janssen following a review of a large number of applications for the panel. The members were selected based on their expertise in mathematics and mathematics instruction. The panel membership represents diversity with respect to organizational affiliation, geographic region, and credentials held. In addition, a consultant from the California Department of Education (CDE) served as a liaison to the panel.

In June 2009, an agenda item (<http://www.ctc.ca.gov/commission/agendas/2009-06/2009-06-6E.pdf>) continued the discussion related to the teaching of mathematics in California. The focus of the item was the pedagogical preparation required for individuals who are authorized to teach mathematics.

The June 2010 Commission agenda item (<http://www.ctc.ca.gov/commission/agendas/2010-06/2010-06-5D.pdf>) included the charge to the TMAP, presented an overview of panel deliberations, and identified the major issues that the panel discussed. The agenda item also provided an update on the work of the TMAP, their proposed changes to the structure of the mathematics specialist authorization, and presented proposed draft program standards for the revised authorizations as well as for the mathematics pedagogy preparation of multiple subject credential candidates.

At the June 2010 TMAP meeting, stakeholder feedback from the survey was discussed. Stakeholder feedback in general was very positive with 80% of the 60 respondents stating that they had no problems with the concepts or proposed language of the standards. The panelists reviewed all comments regarding clarity and made revisions as needed.

Issues Identified in the Preparation of Individuals to Teach Mathematics

Based on the study and discussion of research articles, national panel recommendations, Commission agenda reports, and the California mathematics curriculum framework, the TMAP decided to focus its work in two areas: 1) restructuring and updating the authorizations and standards for the Mathematics Specialist Credential, and 2) expanding the mathematical pedagogy preparation for Multiple Subject credential candidates. These foci were chosen because they provided a mechanism for responding relatively quickly to the critical need for mathematics teaching expertise at the K-8 grade levels (math specialist) and to the long-term needs of K-8 students for mathematically-competent multiple subject teachers.

Overview of Revised Mathematics Specialist Credentials

The TMAP came to agreement that although teachers of mathematics have at least a basic level of knowledge and expertise regarding teaching mathematics, there is a pressing need to provide additional support to teachers by increasing access to mathematics teaching expertise on a consistent basis, especially at the elementary and middle grades levels. The panel felt that access to expertise in mathematics would best be accomplished by updating the former mathematics specialist credential program standards so that more mathematics specialists could be available statewide to support classroom teachers. As the panel refined its thinking about the needed changes and updates to the mathematics specialist credential, it determined that a structure that parallels the current, nested structure of the reading certificate and reading specialist credential would be appropriate in the area of mathematics.

First, the panel proposes revising and renaming the existing authorization as the Mathematics Instructional Certificate (MIC). In considering what knowledge, skills, and experience a MIC holder should have, the panel agreed that an individual with this certificate should be required to complete advanced preparation and fieldwork in both mathematics content and the pedagogy of mathematics above and beyond what is required for the multiple subject teaching credential. In addition, the panel recognized that within the MIC option, some teachers would have the prerequisite math content mastery to obtain a certificate that would go through but not beyond the level of mathematics typically taught in Kindergarten through Pre-Algebra, whereas other teachers might possess the math content that would authorize the teaching of mathematics Kindergarten through Algebra I.

Further, in addition to the MIC, the panel felt that there should be an authorization to recognize a higher level of specialized skills that would allow an individual to not only provide support to teachers, but also provide leadership at the K-12 level with respect to the teaching and learning of mathematics. This proposed credential is the Mathematics Instructional Leadership (MIL) Specialist Credential.

Following is a further description of the MIC and the MIL Specialist credential. The proposed preconditions for these two authorizations are provided in Appendix B.

Mathematics Instruction Certificate (MIC): K-Pre-Algebra and K-Algebra I

Candidates with the prerequisite teaching credential may pursue either of two authorizations through a MIC program, depending on the mathematics content knowledge of the candidate and the authorization desired:

- (1) The MIC (K-Pre-Algebra) would apply to candidates who can document mastery of mathematics knowledge, as organized in the California mathematics content standards, from Kindergarten through Algebra I.
- (2) The MIC (K-Algebra I) would apply to candidates whose mathematics knowledge includes mastery of the California mathematics content standards from Kindergarten through Algebra I, II, and Geometry.

It should be pointed out that the actual knowledge base of the teacher is required to be at a higher level than what the teacher would be authorized to teach.

Structure of the Mathematics Instruction Certificate

MIC Route	Precursor Credential	Mathematic Content Knowledge of the Teacher	Authorizes Teaching
K-Pre-Algebra	Teaching Credential	K-7 through Algebra I	Kindergarten -Pre-Algebra
K-Algebra I		K-7, Algebra I, Geometry, and Algebra II	Kindergarten-Algebra I

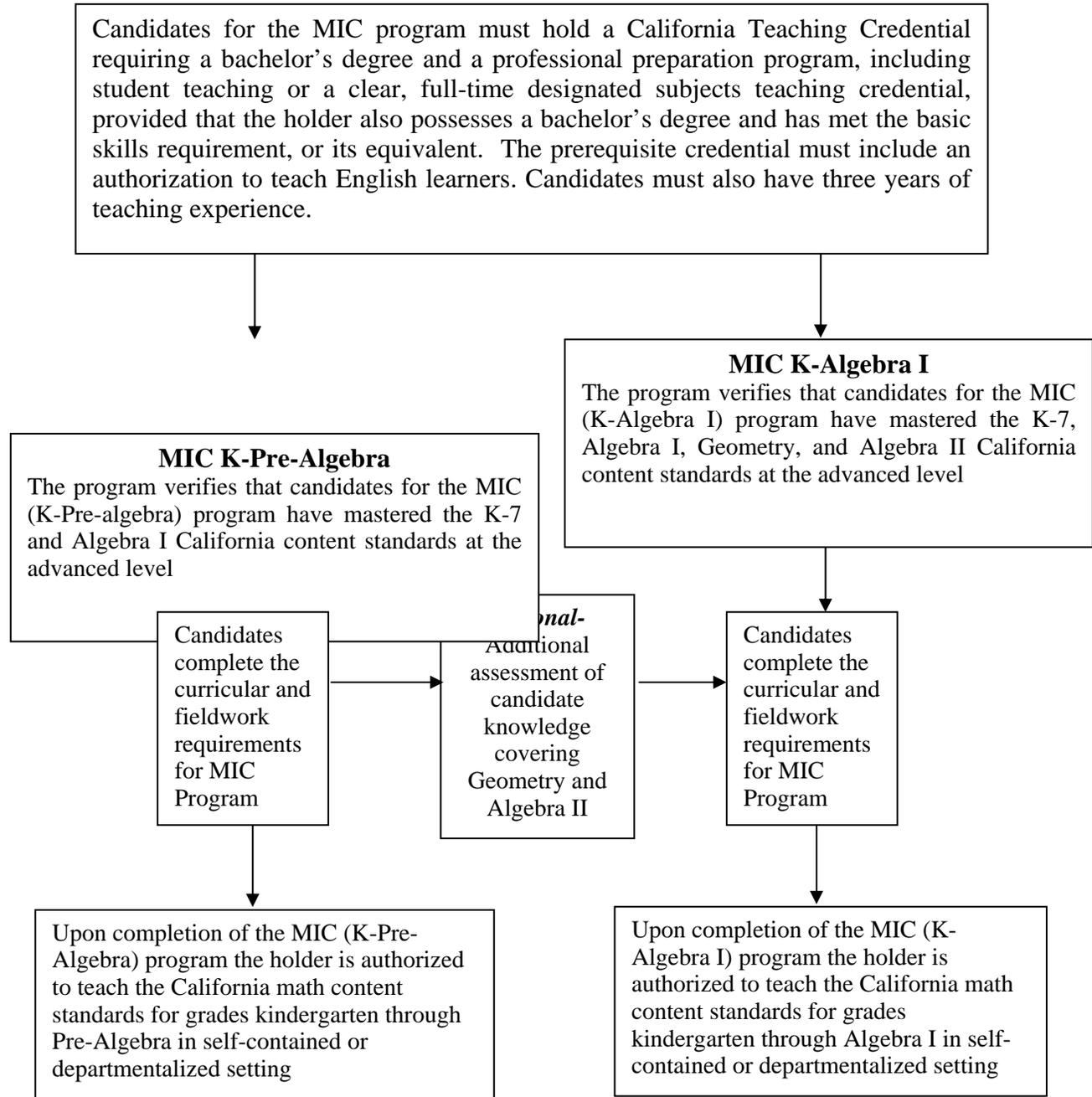
The MIC would be an authorization beyond the preliminary, life, or clear multiple subject teaching credentials. It is anticipated that the holder of either MIC authorization would play a major role in bridging the existing achievement gap as he or she would have expertise in curriculum design, coaching teachers, designing and implementing intensive interventions, and teaching teachers to effectively intervene, accommodate, and differentiate their mathematics instruction to increase student engagement and proficiency in mathematics from Kindergarten through Pre-Algebra or Algebra I.

The holder of either MIC authorization would need to have the knowledge and skills needed to provide leadership in a comprehensive Kindergarten through Pre-Algebra/Algebra I mathematics program that addresses the instructional needs of English learners, students with disabilities, gifted and talented students, and students at risk. Additionally, the holder of either MIC authorization could potentially teach mathematics from Kindergarten through Pre-Algebra/Algebra I in a departmentalized setting. The impact of the holder of either MIC authorization might include, but not be limited to, increasing:

- student proficiency in K-Pre-Algebra/Algebra I and closing the achievement gap by providing math instructional leadership to schools, districts, and counties in areas such as curriculum design, coaching, intensive interventions, accommodation, and differentiation
- expertise in teaching K-Pre-Algebra/Algebra I subject matter in either a departmentalized or self-contained setting to all students, including English learners, students with disabilities, gifted and talented students and students at risk
- the number of highly qualified K-Pre-Algebra/Algebra I teachers in departmentalized settings

The program standards that provide guidance to program sponsors interested in providing preparation for the Mathematics Instruction Certificate are provided in Appendix C.

Figure 1: Structure for the Proposed Mathematics Instruction Certificate (MIC)



Proposed Mathematics Instructional Leadership (MIL) Specialist Credential

Individuals must complete the Mathematics Instruction Certificate program before they would be eligible for the MIL Specialist credential as this credential is built upon the MIC. The proposed new MIL Specialist credential would provide experienced teachers the skills required to:

- promote more effective teaching and learning of mathematics PreK-12
- provide the leadership and a vision for mathematics instruction for schools, districts, and county offices
- fulfill a need in the field for a cadre of mathematics teacher leaders who can connect content level and coaching expertise with school, district, and/or county leadership

Programs preparing MIL Specialist credential candidates would include advanced preparation and fieldwork in:

- effectively connecting action research and mentoring/coaching skills with theoretical research to bridge the theory and practice divide in mathematics teaching and learning
- designing and implementing a school and/or district professional development system that involves teachers and administrators in working collaboratively to increase student engagement and learning in mathematics
- analyzing and using student, school, district, county, state, and college/university data to inform school and district program design to increase the number of students who are college-ready and to reverse the pervasive achievement gap
- leading a professional community of practice

The draft program standards for the proposed MIL Specialist credential are shown in Appendix D.

Improved Preparation to Teach Mathematics for Multiple Subject Teachers

The current standards for multiple subject preparation programs contain one standard that is devoted specifically to the teaching of reading – Program Standard 7A. One result of this emphasis is that teacher preparation programs typically have at least one course that focuses exclusively on developing candidates’ knowledge and skills for teaching reading. In contrast, program standards for preparing candidates’ pedagogical skills in mathematics are found in a standard that also defines the content for subject-specific pedagogical preparation for science, history-social science, the visual and performing arts, physical education, and health.

The panel determined that the current single standard that includes mathematics along with other content areas does not provide enough specificity to ensure that multiple subject candidates develop the mathematical knowledge for teaching that is essential for ensuring that children in K-8 classrooms receive effective instruction in mathematics. To place more emphasis on developing the mathematics knowledge of multiple subject teachers, the panel developed draft language for a mathematics-specific teacher preparation program standard. The standard describes the critical mathematics content knowledge, specialized content knowledge for teaching mathematics, and mathematics pedagogical skills that multiple subject credential holders must have to be effective early teachers of mathematics.

The following excerpt from the proposed draft program standard illustrates this focus:

“Specifically, the program coursework and fieldwork considers three domains of professional knowledge to be central to the work of teaching mathematics: mathematics content knowledge, specialized mathematical knowledge for teaching and thinking, and pedagogical knowledge and practices for teaching mathematics. The specifications for the Multiple Subject CSET provide a basis for documenting candidates’ foundational mathematical content knowledge prior to field experiences. The program develops candidates’ specialized mathematical knowledge for teaching and thinking, as well as pedagogical knowledge and practices for teaching mathematics. The program lays a solid foundation of these three domains of professional knowledge and teaches candidates to use and integrate these domains in their developing practice. “

The draft standard is included in Appendix E and is proposed to replace the current language in Standard 8-A(a) which is provided in Appendix F. Since this recommendation, along with recommendations relating to the teaching of English learners, impacts the Multiple and Single Subject program standards as a whole and has implications for the unit limitation, this standard will be brought separately to the Commission for consideration

Staff Recommendation

Staff recommends that the Commission approve the proposed preconditions and program standards for the Mathematics Instructional Certificate and Mathematics Instructional Leadership Specialist credential as presented in this agenda item.

Next Steps

If the Commission takes action to adopt the proposed preconditions and program standards, staff will develop a standards handbook and disseminate the preconditions and standards to all institutions via a Program Sponsor Alert and the Commission website. These will include a timeline for transition and implementation and a date on which technical assistance will be provided via webcast. In addition, following the approval of the standards, the Commission will start the Title 5 regulatory process and present an information item with proposed regulations for the Mathematics Instructional Certificate and Mathematics Instructional Leadership Specialist credential.

Appendix A

CTC Teaching Mathematics Advisory Panel (2009)

Name	Employer	Representing
CK Green	Newport-Mesa Unified School District	California Federation of Teachers (CFT)
Zulmara Cline	California State University, Office of the Chancellor	California State University, Office of the Chancellor
Jody Priselac	University of California, Los Angeles	University of California, Office of the President
Katharine Clemmer	Loyola Marymount University	The Association of Independent California Colleges and Universities (AICCU)
Pam Tyson	Contra Costa County Office of Education	California County Superintendents Educational Services Association (CCSESA)
Phil Quon	Cupertino Unified School District	Association of California School Administrators (ACSA)
Jan Bridge	Chino Valley Unified School District	California Teachers Association (CTA)
	None appointed	California School Boards Association
Brenda Hensley	Vacaville Unified School District	
Carole Vargas	Folsom Cordova Unified School District	
Katherine Morris	Sonoma State University	
Kyndall Brown	University of California, Los Angeles	
Lisa Hoegerman	Apple Valley Unified School District	
Dennis Parker	University of the Pacific	
Nadine Bezuk	San Diego State University	
Sunny Chin-Look	Alhambra Unified School District	
Vriana Kempster	San Francisco Unified School District	
Zeev Wurman	Independent Consultant	
Megan Holstrom	High Tech High, San Diego	
David Simmons	Ventura County Office of Education	
Michael Fickel	California State University, San Marcos	
Jim Greco	California Department of Education	
Sharen Bertrando	CA Comprehensive Center, WestEd	
Staff Working with the Math Advisory Panel		
Terry Janicki	Commission on Teacher Credentialing	
Rebecca Parker	Commission on Teacher Credentialing	
Teri Clark	Commission on Teacher Credentialing	

Appendix B

Proposed Preconditions Mathematics Instructional Certificate Program and Mathematics Instructional Leadership Specialist Credential Program

Mathematics Instructional Certificate (MIC) Program

1. A program sponsor that operates a program for the Mathematics Instructional Certificate (MIC) shall determine, prior to admission to the credential program, that each candidate possesses a valid California teaching credential requiring a bachelor's degree and a professional preparation program, including student teaching or a clear, full-time designated subjects teaching credential, provided that the holder also possesses a bachelor's degree and has met the basic skills requirement, or its equivalent. The prerequisite credential must include an authorization to teach English learners.
2. A Commission-approved program shall determine prior to recommending a candidate for the MIC, that the candidate has three years of teaching experience.
3. Prior to admission, the program sponsor shall verify the candidate's level of mathematics content expertise. Full admission to the K through Pre-Algebra or the K through Algebra I program is contingent on candidate mastery of the relevant mathematics standards as determined by the program (See elaboration of the required mathematics content knowledge at end of this appendix.)

Mathematics Instructional Leadership (MIL) Specialist Credential Program

1. A Commission-approved program shall determine prior to recommending a candidate for the MIL Specialist credential that the candidate has completed a MIC program.

Elaboration of Mathematics Subject Matter Requirements

The curriculum of the program addresses the Subject Matter Requirements and standards of program quality as set forth in this document. The following is the mathematics background the candidate is expected to know prior to admission to the MIC program.

Subject Matter Domain 1: Algebra

Candidates demonstrate an understanding of the foundations of the algebra contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*. To ensure a clear understanding of the conceptual underpinnings of algebra in elementary and middle school settings, candidates are skilled at symbolic reasoning and use algebraic skills and concepts to model a variety of problem-solving situations.

1.1 Algebraic Structures

- a. Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$)
- b. Know that the rational numbers and real numbers can be ordered, but that any polynomial equation with real coefficients can be solved in the complex field
- c. Know why graphs of linear inequalities are half planes and be able apply this fact (e.g., linear programming)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 1.0, 2.0; Grade 7, Algebra and Functions: 1.0; Algebra I: 1.0, 3.0-7.0, 9.0-15.0, 24.0, 25.0; Geometry: 1.0, 17.0; Algebra II: 1.0-8.0, 11.0, 24.0, 25.0; Trigonometry: 17.0—Students are familiar with complex numbers. They can represent a complex number in polar form, 19.0)

1.2 Polynomial Equations and Inequalities

- a. Prove and use the Quadratic Formula for real and complex quadratic polynomials
- b. Understand and use the proofs of the following:
 - The Division Algorithm
 - The Factor Theorem
 - The Conjugate Roots Theorem for polynomial equations with real coefficients
- c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra

(Mathematics Content Standards for California Public Schools, Grade 7, Algebra and Functions: 2.0-4.0; Algebra I: 1.0, 2.0, 4.0-10.0, 12.0-15.0, 17.0-23.0; Algebra II: 2.0-11.0, 16.0, 17.0)

1.3 Functions

- a. Analyze and derive general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions)
- b. Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 1.0; Grade 7, Number Sense: 1.0, 2.0; Algebra and Functions: 3.0; Algebra I: 3.0-6.0, 10.0, 13.0, 15.0-18.0, 21.0-23.0; Algebra II: 1.0-4.0, 6.0-17.0, 24.0, 25.0; Trigonometry: 2.0, 19.0)

Subject Matter Domain 2: Geometry

Candidates demonstrate an understanding of the foundations of the geometry contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*. To ensure a clear understanding of the conceptual underpinnings of geometry in elementary and middle school settings, candidates understand, apply, and prove theorems relating to a variety of topics in two- and three-dimensional geometry.

2.1 Parallelism

- a. Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees)

(Mathematics Content Standards for California Public Schools, Algebra I: 8.0, 24.0; Geometry: 1.0-3.0, 7.0, 13.0)

2.2 Plane Euclidean Geometry

- a. Prove simple theorems and solve problems involving similarity and congruence (e.g., base angles of isosceles triangles are congruent)
- b. Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, the Pythagorean Theorem and its converse)
- c. Understand, apply, and justify properties of polygons and circles (e.g., derive the area formulas for regular polygons and circles from the area of a triangle)
- d. Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector)
- e. Use techniques in coordinate geometry to prove geometric theorems (e.g., distance formula, midpoint formula)

(Mathematics Content Standards for California Public Schools, Grade 6, Algebra and Functions: 2.0, 3.0; Measurement and Geometry: 2.0; Grade 7, Measurement and Geometry: 1.0-3.0; Algebra I: 8.0, 24.0; Geometry: 1.0-6.0, 8.0-16.0, 18.0-21.0; Algebra II: 16.0, 17.0; Trigonometry: 12.0,13.0—Students know the law of sines and the law of cosines 19.0; Mathematical Analysis: 5.0)

2.3 Three-Dimensional Geometry

- a. Demonstrate an understanding of parallelism and perpendicularity of lines and planes in three dimensions
- b. Understand, apply, and justify properties of three-dimensional objects (e.g., derive the volume and surface area formulas for prisms, pyramids, cylinders)

(Mathematics Content Standards for California Public Schools, Grade 6, Measurement and Geometry: 1.0; Grade 7, Measurement and Geometry: 2.0; Algebra I: 24.0; Geometry: 2.0, 3.0, 12.0, 17.0; Mathematical Analysis: 5.0)

Subject Matter Domain 3: Number Theory

Candidates demonstrate an understanding of the number theory and a command of the number sense contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*. To ensure a clear understanding of the conceptual underpinnings of number theory in elementary and middle school settings, candidates prove and use properties of natural numbers. They formulate conjectures about the natural numbers using inductive reasoning, and verify conjectures with proofs.

3.1 Natural Numbers

- a. Use basic properties of natural numbers (e.g., properties of divisibility)
- b. Use the Principle of Mathematical Induction to prove results in number theory (e.g., the sum of any two odd integers is even)
- c. Apply the Fundamental Theorem of Arithmetic (e.g., find the greatest common factor and the least common multiple, show that every fraction is equivalent to a unique fraction where the numerator and denominator are relatively prime, prove that the square root of any number, not a perfect square number, is irrational)

(Mathematics Content Standards for California Public Schools, Grade 6, Number Sense: 2.0; Grade 7, Number Sense: 1.0; Algebra I: 1.0, 2.0, 12.0, 24.0, 25.0; Geometry: 1.0; Algebra II: 21.0, 23.0, 25.0; Mathematical Analysis: 3.0)

Subject Matter Domain 4: Probability and Statistics

Candidates demonstrate an understanding of the statistics and probability distributions for advanced placement statistics contained in the *Mathematics Content Standards for California Public Schools (1997)* as outlined in the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve (2005)*. To ensure a clear understanding of the conceptual underpinnings of probability and statistics in elementary and middle school settings, candidates solve problems and make inferences using statistics and probability distributions.

4.1 Probability

- a. Understand and apply basic principles of permutations and combinations
- b. Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles)
- c. Use and explain the concept of conditional probability

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 3.0; Algebra II: 18.0-20.0; Probability and Statistics: 1.0-4.0)

4.2 Statistics

- a. Compute and interpret the mean, median, and mode of discrete distributions
- b. Compute and interpret the range of both discrete and continuous distributions
- c. Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results

(Mathematics Content Standards for California Public Schools, Grade 6, Statistics, Data Analysis, and Probability: 1.0, 2.0; Grade 7, Statistics, Data Analysis, and Probability: 1.0; Probability and Statistics: 5.0-7.0)

Appendix C

Mathematics Instructional Certificate (MIC) Proposed Program Standards

Category A: Program Design

Standard 1: Program Design

The preparation program and any prerequisites include a purposeful, interrelated, and developmentally-designed sequence of coursework and field experiences. The design of the program follows an explicit statement of program philosophy and purpose based on a sound rationale informed by theory, research, and practice. It effectively coordinates and articulates expertise in integrating and applying K-Pre-Algebra and/or K-Algebra I content knowledge, specialized mathematical knowledge for teaching and thinking, and pedagogical knowledge and practices for teaching mathematics.

The sponsoring institution demonstrates a commitment to candidate preparation by providing appropriate support for the program. The program has a qualified leadership team with expertise in mathematics content, mathematics education, teacher education, and teacher leadership.

The program provides extensive opportunities for candidates to demonstrate mathematical and pedagogical content knowledge and skills to support effective mathematics instruction and student learning. Coursework and fieldwork address the complex interplay of math content and pedagogy in effective teaching. Candidates are prepared to enhance mathematical development for all students including English learners, students with disabilities, students who are gifted and talented, and students at risk. Candidates are prepared to collaborate and co-teach with other math teachers as well as teachers of other subject matter disciplines. The program includes a planned process of comprehensive assessments ensuring that candidates are prepared to teach K through Pre-Algebra or K through Algebra I. In addition, its design ensures that candidates are equipped to understand the challenges of developing mathematics literacy among California's diverse student and teaching population.

Category B: Curriculum

These three mutually supportive domains are defined by the following seven elements, organized into two standards, which provide structure for the program design:

Mathematical Content Knowledge	Specialized Mathematical Knowledge for Teaching and Thinking	Pedagogical Knowledge and Practices for Teaching Mathematics
Admission to the program is contingent on candidate mastery of mathematics as determined by the program based on the math content outlined in the preconditions.	<ol style="list-style-type: none">1. Students' mathematical thinking2. Mathematical representation3. Mathematical language	<ol style="list-style-type: none">4. Mathematics curriculum5. Instructional planning6. Classroom discourse7. Assessment

Standard 2: Specialized Mathematics Knowledge and Thinking

The program provides opportunities for candidates to develop Mathematical Knowledge for Teaching and understand mathematics content, consistent with the candidate's level of certification. The program provides opportunities for candidates to develop advanced competency to:

1. Plan and implement instruction that includes differentiation, accommodations, and interventions and is based on students' mathematical thinking by:
 - a. Using error analysis processes to review and understand students' misconceptions and to distinguish whether a specific error reflects a misunderstanding of the mathematical process or a need for language development in the area of mathematical concepts
 - b. Understanding the order of presentation of mathematical concepts that lead to student proficiency in mathematics
 - c. Analyzing the learning trajectories of individual students
 - d. Explaining standard and alternative algorithms and solution strategies
2. Use a variety of appropriate methods of mathematical representation, including technology, oral language, written symbols, pictures, concrete materials/models, real-world situations, while also:
 - a. Identifying and understanding strengths and limitations of various representations of mathematical concepts
 - b. Linking representations to underlying mathematical theories and to other representations.
3. Use and understand the correct usage of mathematical language by:
 - a. Analyzing mathematical vocabulary in the context of mathematical concepts when listening and responding to students' questions and comments
 - b. Evaluating their own and colleagues' presentation of mathematical definitions and terms, as well as considering students' culture, language, and cognitive needs when using academic language to scaffold instruction

Standard 3: Pedagogical Knowledge for Teaching Mathematics

The program provides candidates with opportunities to develop advanced practices in the use of students' mathematical thinking, mathematical representation, mathematical language, mathematics curriculum, instructional planning, classroom discourse, and assessment. The program will provide opportunities for candidates to develop advanced skills in evaluating, planning, and implementing appropriate interventions to increase student achievement.

The program will provide opportunities for candidates to develop the type of pedagogical expertise needed to modify curriculum to address the specific needs of diverse groups of students, including but not limited to struggling students, English language learners, gifted and talented students, and students with special needs.

Candidates must be able to demonstrate advanced competency to:

4. Plan and implement mathematics instruction in developmentally and culturally responsive ways to meet specific student needs, including the ability to:
 - a. Analyze and adapt state-adopted curriculum materials for targeted audiences based on students' cultural, linguistic, and cognitive development
 - b. Identify and connect concepts that are fundamental to learning mathematics, such as place value, fractions, real numbers and algebra
5. Plan instruction that supports students' learning of mathematics by:

- a. Selecting and developing student learning tasks that enable teachers to understand the conjectures and generalizations that students make
 - b. Aligning instructional goals, assessments, instructional strategies, and practice (e.g., assignments, homework)
 - c. Designing and implementing flexible grouping strategies (homogeneous, semi-homogeneous, heterogeneous, large group, small group, and individual learning) according to students' needs and level of achievement
 - d. Focusing on the mathematics content standards and the key concepts within the standards
 - e. Collaborating with individual teachers (pre-service, novice, and experienced) through co-planning and co-teaching to improve student learning
6. Develop strategies for classroom discourse by being able to:
- a. Facilitate student to student interaction
 - b. Analyze questioning strategies to lead discussions that actively involve all students
 - c. Select culturally appropriate examples and reframe problems to encourage students' deep understanding within a mathematical context
 - d. Advance and cultivate positive attitudes toward mathematics; encouraging curiosity, flexibility, and persistence in solving mathematical problems
7. Use assessments for:
- a. Identifying gaps in students' knowledge and for designing instruction to bridge the gaps, language assessments for identifying gaps in understanding mathematics terms, summative assessments and standardized assessments for measuring student growth
 - b. Guiding instruction and developing curriculum that is targeted, accommodated, and differentiated for intensive intervention as necessary
 - c. Communicating progress to students, parents, colleagues, and other appropriate service providers
 - d. Deriving demographic, process, and outcome data at the student, school, and district levels to support informed decisions in designing targeted instruction that promotes students equitable access to learn high-level mathematics

Category C: Fieldwork

Standard 4: Field Experiences

Programs facilitate individualized and balanced field experiences that provide candidates with timely and ongoing feedback to guide improvements in practice as described in Category B. These field experiences are integrated into coursework and are aligned with the candidate competence standard.

The guided field experiences extend candidates' understandings of the three domains and their elements. The candidate is provided substantive opportunities to observe and practice each of the proficiencies described in Category B. The fieldwork component will include the following grade spans: Kindergarten through Grade 3 and Grade 4 through Grade 7 for the candidates that are prepared to teach K through Pre-Algebra. In addition, candidates prepared to teach K through Algebra I also have field experience in Algebra 1. The program collaborates with local educational agencies in providing guidance, site-based support, and coordination of field experiences to ensure the candidate has successful experiences working with English learners,

students with disabilities, students who are gifted and talented, and students at risk.

Category D: Candidate Competence

Standard 5: Determination of Candidate Competence

Program sponsors use multiple measures to determine that each candidate has demonstrated competence across the proficiencies described in Category B, including advanced level culminating projects to demonstrate professional competency. Program options for advanced level culminating projects to demonstrate professional competency may include, but are not limited to, professional presentations, action research, designing curriculum, and school, district, or county collaborative projects.

Appendix D

Mathematics Instructional Leadership (MIL) Specialist Proposed Program Standards

Category A: Program Design

Standard 1: Program Design

The preparation programs and their prerequisites include a purposeful, interrelated, developmentally designed sequence of coursework and field experiences. Programs provide integrated coursework and fieldwork through a model that enables each candidate to demonstrate proficiency as a MIL Specialist. The program includes a planned process for the comprehensive assessment of candidates in the following areas:

1. understand practitioner research and encourage teachers to use it in their practice
2. design and implement professional development that engages teachers, administrators, and parents while promoting student engagement and achievement in mathematics
3. analyze and use data to design solutions to the challenges of developing mathematical literacy among California's diverse population
4. lead a professional community of practitioners to promote student engagement and achievement in mathematics and reverse the achievement gap

Category B: Curricular Requirements

Standard 2: Leadership Knowledge and Skills for the Mathematics Instructional Leader:

The candidate will facilitate the use of a variety of appropriate content-based learning materials and learning strategies that recognize students as active learners, understand the importance of reflection and inquiry, emphasize the quality of student application and performance, utilize appropriate and effective technology, and accelerate mathematics achievement for all students, including English Learners, students with special needs, gifted and talented students, and students at risk. Candidates will guide and support the long-term professional development of staff, consistent with the ongoing effort to improve the learning of all students, relative to the content standards, and provide opportunities for all members of the school community to develop and use skills in distributed leadership and shared responsibility. The candidates will utilize multiple assessments, including assessments that are sensitive to the learning needs of special populations in terms of language, culture, language and processing and cognitive difficulties, to evaluate student learning in an ongoing process focused on improving the academic performance of each student.

Specifically, the program prepares candidates to demonstrate expertise in the following four areas: research-supported mathematics teaching, learning and coaching, professional development and learning, using data to inform student instruction and professional development, and developing professional learning communities.

Category C: Fieldwork

Standard 3: Fieldwork Integrated with Coursework for Mathematics Instructional Leader:

Candidates are provided extensive opportunities to observe, acquire, and use appropriate pedagogical content knowledge for teaching, coaching, and mentoring, and to acquire skills to design and implement innovative processes that are research supported, including uses of

technology. Programs provide candidates with timely and on-going feedback to guide improvement in practice through action research connected to instruction, program design, assessment, and leadership. These field experiences are embedded in coursework and aligned with the program assessment standards. The program provides opportunities for candidates to collaborate with local educational agencies in providing guidance, site-based support, and supervision of field experiences.

Programs facilitate individualized and balanced field experiences that provide candidates with timely and ongoing feedback to guide improvements in practice as described in Category B. These field experiences are integrated into coursework and are aligned with the candidate competence standard. Candidates will support opportunities for all members of the school community to develop and use skills in collaboration, distributed leadership, and shared responsibility in ways that are sensitive to students' families' cultures.

MIL Specialist credential candidates must also demonstrate the capacity to analyze the effectiveness of their own practices in terms of the direct impact of their practices on the people with whom they work (e.g., students, teachers, parents, administrators, and community members), in part, by eliminating the achievement gap, and the real or potential impact of their practices on research of students and student learning of mathematics.

Category D: Assessment of Candidate Competence

Standard 4: Determination of Candidate Professional Competence for the Mathematics Instructional Leader:

Program sponsors may provide any combination of advanced level culminating projects through which candidates demonstrate professional expertise and competency that reflect the candidates' capacity to tailor assessment, instruction, and professional support to the needs of all students, including the special needs of students of different cultures, language levels, and with processing and cognitive difficulties. Candidates will collect field-based evidence throughout the program to demonstrate competence in the four areas of leadership practice at various grade spans (Kindergarten-3, 4-7, Algebra I, Geometry, Algebra II, and Advanced Mathematical study). The evidence should be integrated and demonstrate a professional level of proficiency.

Appendix E

Draft Standard 8-A (a): Pedagogical Preparation for Mathematics Content Instruction by Multiple Subject (MS) Candidates

The program teaches candidates to apply the *Teaching Performance Expectations* (TPEs), particularly TPE 1A: Teaching Mathematics in a Multiple Subject Assignment, to the teaching of mathematics by implementing curriculum frameworks, state-adopted academic content standards for students, and adopted curriculum materials. Program coursework and fieldwork provide candidates with opportunities to learn and implement mathematics pedagogy.

Overall, the program design ensures that candidates are able to create a mathematics instructional program that meets the diverse needs of California's student population. The program prepares candidates to teach mathematics using a balanced approach, including computational and procedural skills, conceptual understanding, and problem solving, as outlined in the *Mathematics Framework for California Public Schools, Kindergarten through Grade Twelve*. The program provides opportunities for candidates to develop and implement teaching and learning strategies designed to enable all students, including English Learners, students with disabilities, students who are gifted and talented, and students at risk, to become mathematically proficient in the intertwined strands of adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.

Specifically, the program coursework and fieldwork considers three domains of professional knowledge to be central to the work of teaching mathematics: mathematics content knowledge, specialized mathematical knowledge for teaching and thinking, and pedagogical knowledge and practices for teaching mathematics. The specifications for the Multiple Subject CSET provide a basis for documenting candidates' foundational mathematical content knowledge prior to field experiences. The multiple subject credential preparation program develops candidates' specialized mathematical knowledge for teaching and thinking, as well as pedagogical knowledge and practices for teaching mathematics. The program lays a solid foundation of these three domains of professional knowledge and teaches candidates to use and integrate these domains in their developing practice.

These three mutually supportive domains are defined by the following seven elements that provide structure for the credential program design:

Mathematical Content Knowledge	Specialized Mathematical Knowledge for Teaching and Thinking	Pedagogical Knowledge and Practices for Teaching Mathematics
Multiple Subject CSET topics provide foundation	<ol style="list-style-type: none">1. Students' mathematical thinking2. Mathematical representation3. Mathematical language	<ol style="list-style-type: none">4. Mathematics curriculum5. Instructional planning6. Classroom discourse7. Assessment

The program provides documentary evidence of how it comprehensively and coherently integrates the seven elements across domains to develop and strengthen candidate competencies in mathematics teaching.

Below each element are examples of topics programs may use to illustrate depth and coherence (the examples are not intended to be used as a checklist but rather are types of evidence programs may provide):

1. Students' mathematical thinking:
 - a. Use and analyze student work to identify misconceptions and conduct error analysis
 - b. Understand the progression of mathematical concepts that lead to student proficiency in mathematics
 - c. Explore and compare standard and alternative algorithms and solution strategies
2. Mathematical representation:
 - a. Use a variety of modes of representation (oral language, written symbols, pictures, concrete materials/models, real-world situations) for mathematics concepts
 - b. Recognize benefits and drawbacks of the various modes of representation
 - c. Link representations to underlying mathematical theories and to other representations
3. Mathematical language:
 - a. Effectively use the academic language of mathematics when referring to or explaining mathematical concepts
 - b. Understand the challenges to student understanding posed by the use within mathematics teaching and learning of words that may have different meanings in contexts outside of mathematics
4. Mathematics curriculum:
 - a. Review, analyze, and sequence state-adopted curriculum materials as appropriate to student learning needs
 - b. Identify, teach, and connect to each other key math concepts, such as place value, fractions, and number sense
5. Instructional planning:
 - a. Provide students with tasks that require or facilitate students' making of appropriate conjectures and/or generalizations within the context of mathematical thinking and application
 - b. Develop aligned instructional goals, instructional strategies, practice (assignments, homework), and assessments
 - c. Plan for differentiation according to students' needs
 - d. Be familiar with district curriculum resources such as pacing guides or other support materials
 - e. Use scaffolding to promote learning when concepts and skills are introduced to students to assist them in learning mathematics
6. Classroom discourse:
 - a. Use questioning strategies to guide discussions
 - b. Select examples and problems to promote deeper understanding
 - c. Foster positive attitudes toward mathematics and encourage student curiosity, flexibility, and persistence in solving mathematics problems
 - d. Facilitate student-to-student interaction

7. Assessment:
 - a. Use formative, summative, standardized, and authentic assessments
 - b. Use assessment results to plan and adapt instruction

Appendix F

Adopted Multiple Subject Standard Language

8-A(a): Pedagogical Preparation for Subject-Specific Content Instruction by Multiple Subject (MS) Candidates

8-A(a) **Mathematics.** During interrelated activities in program coursework and fieldwork, MS candidates learn specific teaching strategies that are effective in supporting them to teach the state-adopted academic content standards for students in mathematics (K-8). They enable students to understand basic mathematical computations, concepts, and symbols; to use these tools and processes to solve common problems; and to apply them to novel problems. They help students understand different mathematical topics and make connections among them. Candidates help students solve real-world problems using mathematical reasoning and concrete, verbal, symbolic, and graphic representations. They provide a secure environment for taking intellectual risks and approaching problems in multiple ways. Candidates model and encourage students to use multiple ways of approaching mathematical problems, and encourage discussion of different solution strategies. They foster positive attitudes toward mathematics, and encourage student curiosity, flexibility, and persistence in solving mathematical problems.