# 2D

## Action

*Professional Services Committee*

## Program Approval and Initial Institutional Approval

**Executive Summary:** This agenda item presents three single subject matter programs for Commission approval and one prospective program sponsor for initial institutional approval.

**Recommended Action:** That the Commission approve the three single subject matter programs described in this agenda item and the one request for initial institutional approval.

**Presenter:** Larry Birch, Director, Professional Services Division

**Strategic Plan Goal: 1**

**Promote educational excellence through the preparation and certification of professional educators.**

- Sustain high quality standards for the preparation and performance of professional educators and for the accreditation of credential programs.

# Program Approval and Initial Institutional Approval

**Introduction**
This agenda item presents three single subject matter programs submitted by institutions of higher education for single subject matter program approval and one request by a prospective program sponsor for initial institutional approval.

## I. Recommendation for Approval of Single Subject Matter Programs

**Background**
The Commission regularly receives recommendations for program approval from single subject matter review panels. These panels of subject matter experts review all program documentation and make an informed determination as to whether the program meets the standards common to all subject matter programs and the subject specific subject matter standards. The subject specific subject matter standards are closely aligned to the K-12 academic standards. These subject matter programs are usually undergraduate courses of study completed before candidates begin teacher preparation programs. However, in some cases they are completed concurrently with teacher preparation programs.

**Subject Matter Program Review Procedures**
Following are the general procedures for the review of subject matter programs:

1. Technical Assistance – After the Commission adopts a set of new program standards, Commission staff members provide technical assistance to prospective program sponsors wishing to submit responses to the new standards. Technical assistance materials are provided on the Commission's website. Staff members train, assign, and coordinate review team work.

2. Preconditions Review – After the program proposal is received, Commission staff review the sponsor's response to the preconditions. The preconditions are based on both state laws and Commission policies, and address minimum unit and content area requirements. If the preconditions response is incomplete, the sponsor is requested to provide specific information necessary for compliance with the preconditions.

3. Program Review –The program sponsor's responses to the Commission's subject matter program standards are reviewed by a team of two or more subject matter educators to determine if the program meets the program standards, including the subject matter requirements (SMRs). The SMRs are the content knowledge required to be covered in the program and are aligned to the K-12 content standards that the candidate will be expected to know. The reviewers are

trained in the alignment of the standards and subject matter requirements and in the review process before they are assigned proposals to review. Reviewers are instructed to find explicit evidence that programs not only align with K-12 content standards but also introduce their candidates to those standards within the context of their subject matter studies. The team must reach consensus that each standard is met based upon evidence provided in the document. If the program does not meet the standards, the sponsor is given an explanation of the findings. The sponsor may then submit the additional information requested. Once reviewers determine that the program proposal provides a convincing and adequate body of evidence to meet the Commission's adopted subject matter program standards, the program is recommended to the Commission for approval.

4. After subject matter program approval is granted by the Commission, the institution may accept candidates in the approved subject matter program. Graduates of a Commission approved single subject matter preparation program meet the Commission's subject matter requirement and are not required to take the subject matter examination (CSET).

This report presents three single subject matter programs which have been deemed to have met all of the appropriate *Standards of Quality and Effectiveness for Single Subject Matter Preparation Programs* (www.ctc.ca.gov/educator-prep/STDS-subject-matter.html) by the appropriate review panel and are recommended to the Commission for approval. These three are:

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| California State University, East Bay | Mathematics |
| San Francisco State University: | Mathematics |
| Vanguard University: | Mathematics |

**Summary Information on the Single Subject Matter Programs**

California State University, East Bay: Mathematics (This is the first subject matter program, submitted after January 2007, brought to the Commission for approval that is required to include a matrix showing the alignment of the proposed program to the adopted KSAs and the adopted Content Standards. A copy of the matrix in included as Appendix A.)

The Subject Matter Preparation Program in Mathematics (SMPM) is designed to provide a deep understanding of the mathematics specified in the state mathematics content standards and the mathematics curriculum framework and beyond. Students who successfully complete the SMPM will be well prepared to teach the content in the K-7 strands and the disciplinary content covered in the 8-12 standards. To ensure that students have a strong background in the subjects they will be teaching, the SMPM includes courses that prepare students mathematically in the areas of algebra, geometry, trigonometry, mathematical analysis, linear algebra, probability, statistics and calculus. The program is designed with sufficient additional coursework to ensure that the students have competency and knowledge substantially beyond that expected in the courses taught in the high school.

The Department of Mathematics and Computer Science at California State University, East Bay is committed to providing an academically rigorous and intellectually challenging Subject Matter Program in Mathematics (SMPM). The goal is that students who complete the program will have:

- been equipped with a solid background in the mathematics they will be expected to teach in California schools as specified in the state content standards and framework
- had field experience in a local high school math classroom which prepares them for teaching in a diverse classroom setting
- gained a sufficient understanding of mathematics to enable them to continue to learn new mathematics and to respond to students with special needs and future curricular changes
- been exposed to a variety of models of instruction which accommodate diverse learning styles
- developed a lifelong enthusiasm for mathematics and other intellectual activity, including the integration of knowledge and the application of scholarship to practical problems
- experienced communicating mathematics verbally and in written form through formal presentations and papers and through informal discussions

<u>San Francisco State University: Mathematics</u>

The underlying philosophy of the proposed program is to provide prospective mathematics teachers with the subject matter preparation needed to teach effectively consistent with current State requirements. The program, which includes rigorous coursework and practical field experience, emphasizes mathematical reasoning and problem solving, connections among mathematical concepts, technology, and communication skills. Mathematics education faculty from the Department of Mathematics and the Department of Secondary Education designed the program after considering the needs of prospective mathematics teachers as reflected in the *Mathematics Content Standards for California Public Schools: Kindergarten Through Grade Twelve*, the *Mathematics Framework for California Public Schools: Kindergarten Through Grade Twelve*, the Subject Matter Requirements in mathematics, and the Standards of Program Quality and Effectiveness. Prospective mathematics teachers who complete the program will have a strong foundation of subject matter knowledge, be able to use mathematical and statistical software as a tool for understanding concepts and solving problems, and be able to communicate mathematics effectively both orally and in writing.

The program coursework enables prospective mathematics teachers to learn core concepts from an advanced viewpoint. The program coursework requires prospective mathematics teachers to learn mathematics in a variety of ways, explore connections between mathematical concepts, and develop analytical skills. Prospective mathematics teachers in the program develop their mathematical reasoning skills, explore connections among branches of mathematics, and have opportunities for problem solving and mathematical communication. In the program, candidates will:

- obtain a strong foundation of mathematics knowledge,
- use mathematics and statistical software as a tool for understanding concepts and solving problems, and
- learn to communicate mathematics effectively both orally and in writing.

Vanguard University: Mathematics

Students in the Vanguard University Mathematics Subject Matter Program receive mathematical training to prepare them for careers in elementary or secondary education. The basic theories and principles of mathematics are studied as well as their applications. Students are required to take a full year of calculus-based physics. While lower division coursework is designed to develop the foundations of mathematics, the upper division coursework extends the subject matter learning and prepares students for continued development during professional teacher preparation and induction programs.

The instructional formats and uses of technology in the Vanguard University Mathematics Program were selected for the appropriate content taught at the different grade levels in California middle and secondary schools, including algebra, geometry, history of mathematics, number theory, statistics and probability, and calculus. The varied instructional strategies model for prospective teachers the many possibilities for instruction in K-12 schools.

The philosophy of the Subject Matter Program has the express purpose to prepare students for an academic teaching career. Candidates are prepared to teach mathematics so they can demonstrate the following qualities of effective teaching: effectual communication skills, good attitude toward students, knowledge of subject (conceptually, historically, and for application), strong organization, enthusiasm; fairness, flexibility, ability to encourage students to think for themselves, ability to stimulate learning experiences. The program seeks to develop teachers who are:

- practicing qualified mathematicians
- learning guides enhancing student's mathematical curiosity
- socializing agents for mathematical interaction
- facilitators promoting mathematics learning
- role models expressing excitement in the mathematical field
- persons possessing high morals and ethics

## II. Recommendation for Initial Institutional Approval

**Background**

Prior to 1995, institutions not previously approved to offer programs of professional preparation would submit a program proposal responding to the preconditions and standards of the Commission on Teacher Credentialing. If the institution was accredited by the Western Association of Schools and Colleges (WASC) or another regional accrediting body and if the response to the preconditions and standards was judged to be satisfactory, the Commission voted to give approval to the institution to begin offering one or more educator preparation programs.

With the adoption of the *Accreditation Framework* in 1995, the Commission made a distinction between "initial approval of institutions" and "initial approval of programs," as described below.

**Policies for Initial Approval of Institutions**

Pursuant to California Education Code, the Commission has the authority to determine the eligibility of institutions to offer educator preparation programs and to recommend issuance of credentials to candidates completing programs of preparation. This authority also applies to other program sponsors such as school districts, who were made eligible to sponsor professional educator preparation programs through subsequent legislation.

> **Education Code Section 44227 (a) –** The Commission may approve any institution of higher education whose teacher education program meets the standards prescribed by the Commission, to recommend to the Commission the issuance of credentials to persons who have successfully completed those programs.
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> **Education Code Section 44372** – The powers and duties of the Commission on Teacher Credentialing regarding the accreditation system shall include the following:
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> **(c)** Rule on the eligibility of an applicant for accreditation when the applying institution has not previously prepared educators for state certification in California, pursuant to subdivision (a) of Section 44227.

**Adopted Procedures for Initial Institutional Approval and Initial Program Approval**

A prospective program sponsor that wishes to offer a credential program in California and that has not previously been declared eligible to offer a credential preparation program must undergo a two stage initial approval process: 1) initial institutional approval, and 2) initial approval of programs as described below. The steps in the Commission's accreditation system are:

1) Initial Institutional Approval: The institution (program sponsor) that has not previously been declared eligible to offer credential preparation programs must submit an application to the Commission for initial institutional approval. The institution prepares a complete program proposal that responds to all preconditions, Common Standards and appropriate Program Standards. The application is reviewed for compliance with the appropriate preconditions (regional accreditation [or governing board approval], identification of the position responsible for oversight, non-discrimination procedures, completion of a needs assessment, involvement of practitioners in the design of the program, agreement to provide information to the Commission, etc.) and appropriate Common Standards relating to capacity to offer a preparation program. Once the review has been accomplished, a recommendation is brought before the Commission for *initial institutional approval*. This stage determines only an institution's *eligibility* to offer an approved program.

2) Initial Approval of Programs: Once the Commission acts favorably on institutional approval, the *program* proposal is forwarded to the Committee on Accreditation for action. The program sponsor is required to respond to the appropriate credential program standards for each program the institution wishes to offer. These responses are reviewed by a panel of expert advisors, or in some cases, Commission staff, to determine the sufficiency of the responses. Once it is determined that the program proposal meets the

Commission's program standards, the program sponsor is recommended to the Committee on Accreditation for *initial program approval*.

Once granted initial program approval, the institution is given authority to begin the program and will then come under the Commission's continuing accreditation procedures.

**Hebrew Union College – DeLeT – Day School Leadership through Teaching**

Hebrew Union College wishes to operate a California teaching credential preparation program and has applied for initial institutional approval. A review of the application indicates that they have met the requirements for this initial step. The Hebrew Union College – Jewish Institute of Religion (HUC-JIR) Los Angeles School occupies a five-acre site in the center of the city, adjacent to the University of Southern California. It is the academic center established in 1954 to strengthen and provide leadership for the proliferation of Jewish communities and Reform congregations throughout the Western United States. The HUC-JIR is accredited by the Western Association of Schools and Colleges. With its Rhea Hirsch School of Education (founded in 1970 to train educational leadership for the Reform Movement) and School of Jewish Communal Service (founded in 1968 as the first professional school of Jewish communal service in the United States), the Los Angeles School serves as the Reform Movement's location for advanced graduate study in Jewish education and Jewish communal service. Students from these two professional schools have served hundreds of Reform congregations and Jewish communal organizations and agencies as part of their clinical training and internships.

DeLeT – Day School Leadership through Teaching, as proposed, is a teacher preparation program conducted by the Rhea Hirsch School of Education of Hebrew Union College – Jewish Institute of Religion. DeLeT, the Hebrew word for door, provides a doorway into a teaching career through a 13-month program consisting of two summers of study at the College-Institute, a year-long 4-day-per-week placement in an elementary school classroom under the guidance of a mentor teacher, and one-day-per-week of seminars. Teacher candidates would be observed daily in their classrooms by their mentor teachers and weekly by a "Clinical Educator" (field supervisor) sent by DeLeT.

Candidates for admission must hold a BA degree from an accredited college or university. Once accepted, students begin in the summer and then in weekly seminars they study reading and language, math, social studies, science, health, physical education, day school and society, diversity, and child development, and participate in a year-long Teaching and Learning Seminar. The Teaching Performance Expectations are embedded in the coursework throughout the year. Students also conduct a series of seven (7) inquiries throughout the year which engage them in observing individual students and classroom learning communities as well as teaching individual lessons, units, and integrated units.

**Recommendations**
Based upon a determination by reviewers that the following entities have met all relevant standards and requirements, staff recommends Commission approval of the following:

Single Subject Matter Programs

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| California State University, Easy Bay: | Mathematics |
| San Francisco State University: | Mathematics |
| Vanguard University: | Mathematics |

Initial Institutional Approval
Hebrew Union College

# Appendix A

## Subject Matter Alignment Matrix

## for

## California State University, East Bay

| K-12 Standard<br>Domain/Strand | Subject Matter Requirements (SMRs) | Course #s (Include key assignments or assessments, titles of texts, or other evidence that the program meets the SMRs) |
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| **ALGEBRA 1** Symbolic reasoning and calculations with symbols are central in algebra. Through the study of algebra, a student develops an understanding of the symbolic language of mathematics and the sciences. In addition, algebraic skills and concepts are developed and used in a wide variety of problem-solving situations. | | |
| 1.0 Students identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable: | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| 1.1 Students use properties of numbers to demonstrate whether assertions are true or false. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.1.c Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field | Mathy 1130, 3121, 3300, 4901 Appendix 0 and 11- Domain 1 |
| 2.0 Students understand and use such operations as taking the opposite, finding the reciprocal, taking a root, and raising to a fractional power. They understand and use the rules of exponents | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 3.0 Students solve equations and inequalities involving absolute values. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 4.0 Students simplify expressions before solving linear equations and inequalities in one variable, such as 3(2x-5) + 4(x-2) = 12. | 1.2.a Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) | Math 1130, 1304, 2304 Appendix 0 and 11 - Domain 1 |

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| | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 5.0 Students solve multistep problems, including word problems, involving linear equations and linear inequalities in one variable and provide justification for each step. | 1.2.a Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) | Math 1130, 1304, 2304 Appendix 0 and 11 - Domain 1 |
| | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 6.0 Students graph a linear equation and compute the x- and y-intercepts (e.g., graph C262x + 6y = 4). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by (2x + 6y < 4). | 1.2.a Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) | Math 1130, 1304, 2304 Appendix 0 and 11 - Domain 1 |
| | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 7.0 Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations by using the point-slope formula. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 8.0 Students understand the concepts of parallel lines and perpendicular lines and how those slopes are related. Students are able to find the equation of a line perpendicular to a given line that passes through a given point. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 9.0 Students solve a system of two linear equations in two variables algebraically and are able to interpret the answer graphically. Students are able to solve a system of two linear inequalities in two variables and to sketch the solution sets. | 1.2.a Know why graphs of linear inequalities are half planes and be able to apply this fact (e.g., linear programming) | Math 1130, 1304, 2304 Appendix 0 and 11 - Domain 1 |
| | 1.4.b Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |

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| | 1.4.c Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | Math 2101 Appendix 0 and 11 - Domain 1 |
| 10.0 Students add, subtract, multiply, and divide monomials and polynomials. Students solve multistep problems, including word problems, by using these techniques | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 11.0 Students apply basic factoring techniques to second- and simple third-degree polynomials. These techniques include finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials. | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 12.0 Students simplify fractions with polynomials in the numerator and denominator by factoring both and reducing them to the lowest terms. | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 13.0 Students add, subtract, multiply, and divide rational expressions and functions. Students solve both computationally and conceptually challenging problems by using these techniques. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 14.0 Students solve a quadratic equation by factoring or completing the square. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |

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| 15.0 Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 16.0 Students understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions. | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| 17.0 Students determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 18.0 Students determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function and justify the conclusion. | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| 19.0 Students know the quadratic formula and are familiar with its proof by completing the square. | 1.2.b Prove and use the following:• The Rational Root Theorem for polynomials with integer coefficients• The Factor Theorem• The Conjugate Roots Theorem for polynomial equations with real coefficients• The Quadratic Formula for real and complex quadratic polynomials• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| 20.0 Students use the quadratic formula to find the roots of a second-degree polynomial and to solve quadratic equations. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| 21.0 Students graph quadratic functions and know that their roots are the x-intercepts. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |

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| 22.0 Students use the quadratic formula or factoring techniques or both to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 23.0 Students apply quadratic equations to physical problems, such as the motion of an object under the force of gravity. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| Students use and know simple aspects of a logical argument: | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| 24. 0Students explain the difference between inductive and deductive reasoning and identify and provide examples of each. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.4.b Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| | 3.1.b Use the Principle of Mathematical Induction to prove results in number theory | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 24.2 Students identify the hypothesis and conclusion in logical deduction. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |

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| | 1.4.b Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| | 3.1.a Prove and use basic properties of natural numbers (e.g., properties of divisibility) | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 24.3 Students use counterexamples to show that an assertion is false and recognize that a single counterexample is sufficient to refute an assertion. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.4.b Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| | 3.1.a Prove and use basic properties of natural numbers (e.g., properties of divisibility) | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 25.0 Students use properties of the number system to judge the validity of results, to justify each step of a procedure, and to prove or disprove statements: | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 3.1.a Prove and use basic properties of natural numbers (e.g., properties of divisibility) | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 25.1 Students use properties of numbers to construct simple, valid arguments (direct and indirect) for, or formulate counterexamples to, claimed assertions. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 3.1.a Prove and use basic properties of natural numbers (e.g., properties of divisibility) | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 25.2 Students judge the validity of an argument according to whether the properties of the real number system and the order of operations have been applied correctly at each step. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |

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| 25.3 Given a specific algebraic statement involving linear, quadratic, or absolute value expressions or equations or inequalities, students determine whether the statement is true sometimes, always, or never. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| **GEOMETRY** The geometry skills and concepts developed in this discipline are useful to all students. Aside from learning these skills and concepts, students will develop their ability to construct formal, logical arguments and proofs in geometric settings and problems. | | |
| 1.0 Students demonstrate understanding by identifying and giving examples of undefined terms, axioms, theorems, and inductive and deductive reasoning. | 2.2.a Prove theorems and solve problems involving similarity and congruence<br>2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse)<br>2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1304, 1305, 3215 Appendix 0 and 11 - Domain 2 |
| 2.0 Students write geometric proofs, including proofs by contradiction. | | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 3.0 Students construct and judge the validity of a logical argument and give counterexamples to disprove a statement. | | Math 1300, 3215 Appendix 0 and 11 |
| 4.0 Students prove basic theorems involving congruence and similarity. | 2.2.a Prove theorems and solve problems involving similarity and congruence | Math 1304, 1305, 3215 Appendix 0 and 11 - Domain 2 |
| 5.0 Students prove that triangles are congruent or similar, and they are able to use the concept of corresponding parts of congruent triangles. | 2.2.a Prove theorems and solve problems involving similarity and congruence<br>2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1304, 1305, 3215 Appendix 0 and 11 - Domain 2 |
| 6.0 Students know and are able to use the triangle inequality theorem. | | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 7.0 Students prove and use theorems involving the properties of parallel lines cut by a transversal, the properties of quadrilaterals, and the properties of circles. | 2.1.a Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) | Math 3215 Appendix 0 and 11 - Domain 2 |

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| | 2.1.b Know that variants of the Parallel Postulate produce non-Euclidean geometries (e.g., spherical, hyperbolic) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 8.0 Students know, derive, and solve problems involving the perimeter, circumference, area, volume, lateral area, and surface area of common geometric figures. | 2.2.c Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 9.0 Students compute the volumes and surface areas of prisms, pyramids, cylinders, cones, and spheres; and students commit to memory the formulas for prisms, pyramids, and cylinders. | 2.3.b Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres) | Math 1305, 2304 Appendix 0 and 11 - Domain 2 |
| 10.0 Students compute areas of polygons, including rectangles, scalene triangles, equilateral triangles, rhombi, parallelograms, and trapezoids. | 2.2.c Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 11.0 Students determine how changes in dimensions affect the perimeter, area, and volume of common geometric figures and solids. | 2.2.c Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 12.0 Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems. | 2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 13.0 Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles. | 2.2.c Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 14.0 Students prove the Pythagorean theorem. | 2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 15.0 Students use the Pythagorean theorem to determine distance and find missing lengths of sides of right triangles. | | |
| 16.0 Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors, and the line parallel to a given line through a point off the line. | 2.1.a Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) | Math 3215 Appendix 0 and 11 - Domain 2 |

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| | 2.2.d Justify and perform the classical constructions (e.g., angle bisector, perpendicular bisector, replicating shapes, regular n-gons for n equal to 3, 4, 5, 6, and 8) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 17.0 Students prove theorems by using coordinate geometry, including the midpoint of a line segment, the distance formula, and various forms of equations of lines and circles. | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |
| 18.0 Students know the definitions of the basic trigonometric functions defined by the angles of a right triangle. They also know and are able to use elementary relationships between them. For example, tan(x) = sin(x)/cos(x), (sin(x))2 + (cos(x))2 = 1. | 5.1.a Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2x + cos2x = 1 and that this identity leads to 1 + tan2x = sec2x and 1 + cot2x = csc2x | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |
| 19.0 Students use trigonometric functions to solve for an unknown length of a side of a right triangle, given an angle and a length of a side. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 20.0 Students know and are able to use angle and side relationships in problems with special right triangles, such as 30°, 60°, and 90° triangles and 45°, 45°, and 90° triangles. | 2.1.a Know the Parallel Postulate and its implications, and justify its equivalents (e.g., the Alternate Interior Angle Theorem, the angle sum of every triangle is 180 degrees) | Math 3215 Appendix 0 and 11 - Domain 2 |
| | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 21.0 Students prove and solve problems regarding relationships among chords, secants, tangents, inscribed angles, and inscribed and circumscribed polygons of circles. | 2.2.c Understand, apply, and justify properties of polygons and circles from an advanced standpoint (e.g., derive the area formulas for regular polygons and circles from the area of a triangle) | Math 3215 Appendix 0 and 11 - Domain 2 |
| 22.0 Students know the effect of rigid motions on figures in the coordinate plane and space, including rotations, translations, and reflections. | 2.4.a Demonstrate an understanding of the basic properties of isometries in two- and three-dimensional space (e.g., rotation, translation, reflection) | Math 2101, 3215 Appendix 0 and 11 - Domain 2 |
| **ALGEBRA II** This discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system. | | |
| 1.0 Students solve equations and inequalities involving absolute value. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |

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| 2.0 Students solve systems of linear equations and inequalities (in two or three variables) by substitution, with graphs, or with matrices. | 1.4.c Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | Math 2101 Appendix 0 and 11 - Domain 1 |
| 3.0 Students are adept at operations on polynomials, including long division. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| 4.0 Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| 5.0 Students demonstrate knowledge of how real and complex numbers are related both arithmetically and graphically. In particular, they can plot complex numbers as points in the plane. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if $a < b$ and $c < 0$, then $ac > bc$) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.1.c Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field | Mathy 1130, 3121, 3300, 4901 Appendix 0 and 11- Domain 1 |

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| 6.0 Students add, subtract, multiply, and divide complex numbers. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.1.c Know that the rational numbers and real numbers can be ordered and that the complex numbers cannot be ordered, but that any polynomial equation with real coefficients can be solved in the complex field | Mathy 1130, 3121, 3300, 4901 Appendix 0 and 11- Domain 1 |
| 7.0 Students add, subtract, multiply, divide, reduce, and evaluate rational expressions with monomial and polynomial denominators and simplify complicated rational expressions, including those with negative exponents in the denominator. | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 8.0 Students solve and graph quadratic equations by factoring, completing the square, or using the quadratic formula. Students apply these techniques in solving word problems. They also solve quadratic equations in the complex number system. | 1.2.c Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| 9.0 Students demonstrate and explain the effect that changing a coefficient has on the graph of quadratic functions; that is, students can determine how the graph of a parabola changes as a, b, and c vary in the equation y = a(x-b)2 + c. | 1.2.b Prove and use the following:• The Rational Root Theorem for polynomials with integer coefficients• The Factor Theorem• The Conjugate Roots Theorem for polynomial equations with real coefficients• The Quadratic Formula for real and complex quadratic polynomials• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| 10.0 Students graph quadratic functions and determine the maxima, minima, and zeros of the function. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| 11.0 Students prove simple laws of logarithms. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |

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| 11.1 Students understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 11.2 Students judge the validity of an argument according to whether the properties of real numbers, exponents, and logarithms have been applied correctly at each step. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 12.0 Students know the laws of fractional exponents, understand exponential functions, and use these functions in problems involving exponential growth and decay. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 13.0 Students use the definition of logarithms to translate between logarithms in any base. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 14.0 Students understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 15.0 Students determine whether a specific algebraic statement involving rational expressions, radical expressions, or logarithmic or exponential functions is sometimes true, always true, or never true. | 1.3.c Analyze properties of exponential and logarithmic functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3331 Appendix 0 and 11 - Domain 1 |
| 16.0 Students demonstrate and explain how the geometry of the graph of a conic section (e.g., asymptotes, foci, eccentricity) depends on the coefficients of the quadratic equation representing it. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11- Domain 1 |
| | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |

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| 17.0 Given a quadratic equation of the form ax2 + by2 + cx + dy + e = 0, students can use the method for completing the square to put the equation into standard form and can recognize whether the graph of the equation is a circle, ellipse, parabola, or hyperbola. Students can then graph the equation. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |
| 18.0 Students use fundamental counting principles to compute combinations and permutations. | 4.1.a Prove and apply basic principles of permutations and combinations | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 19.0 Students use combinations and permutations to compute probabilities. | 4.1.a Prove and apply basic principles of permutations and combinations | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| | 4.1.b Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles) | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 20.0 Students know the binomial theorem and use it to expand binomial expressions that are raised to positive integer powers. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 21.0 Students apply the method of mathematical induction to prove general statements about the positive integers. | 3.1.b Use the Principle of Mathematical Induction to prove results in number theory | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 22.0 Students find the general term and the sums of arithmetic series and of both finite and infinite geometric series. | 5.5.a Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) | Math 1305, 2304, 3300, Appendix 0 and 11 - Domain 5 |
| 23.0 Students derive the summation formulas for arithmetic series and for both finite and infinite geometric series. | 5.5.a Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) | Math 1305, 2304, 3300, Appendix 0 and 11 - Domain 5 |

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| 24.0 Students solve problems involving functional concepts, such as composition, defining the inverse function and performing arithmetic operations on functions. | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| 25.0 Students use properties from number systems to justify steps in combining and simplifying functions. | 1.1.b Apply basic properties of real and complex numbers in constructing mathematical arguments (e.g., if a < b and c < 0, then ac > bc) | Math 1304, 3000, 3121, 3300 Appendix 0 and 11 - Domain 1 |
| | 1.3.a Analyze and prove general properties of functions (i.e., domain and range, one-to-one, onto, inverses, composition, and differences between relations and functions) | Math 1130, 1300, 1304, 2101, 3000, 3121,3300, 4901 Appendix 0 and 11 - Domain 1 |
| **TRIGONOMETRY** Trigonometry uses the techniques that students have previously learned from the study of algebra and geometry. The trigonometric functions studied are defined geometrically rather than in terms of algebraic equations. Facility with these functions as well as the ability to prove basic identities regarding them is especially important for students intending to study calculus, more advanced mathematics, physics and other sciences, and engineering in college. | | |
| 1.0 Students understand the notion of angle and how to measure it, in both degrees and radians. They can convert between degrees and radians. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 2.0 Students know the definition of sine and cosine as y- and x-coordinates of points on the unit circle and are familiar with the graphs of the sine and cosine functions. | 5.1.b Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle) | Math 1300, 1304 Appendix 0 and 11- Domain 5 |
| 3.0 Students know the identity cos2(x) + sin2(x) = 1: | 5.1.a Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2x + cos2x = 1 and that this identity leads to 1 + tan2x = sec2x and 1 + cot2x = csc2x | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |
| 3.1 Students prove that this identity is equivalent to the Pythagorean theorem (i.e., students can prove this identity by using the Pythagorean theorem and, conversely, they can prove the Pythagorean theorem as a consequence of this identity). | 5.1.a Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2x + cos2x = 1 and that this identity leads to 1 + tan2x = sec2x and 1 + cot2x = csc2x | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |
| 3.2 Students prove other trigonometric identities and simplify others by using the identity cos2(x) + sin2(x) = 1. For example, students use this identity to prove that sec2(x) = tan2(x) + 1. | 5.1.a Prove that the Pythagorean Theorem is equivalent to the trigonometric identity sin2x + cos2x = 1 and that this identity leads to 1 + tan2x = sec2x and 1 + cot2x = csc2x | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |

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| 4.0 Students graph functions of the form f(t) = A sin (Bt + C) or f(t) = A cos (Bt + C) and interpret A, B, and C in terms of amplitude, frequency, period, and phase shift. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 5.0 Students know the definitions of the tangent and cotangent functions and can graph them. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 6.0 Students know the definitions of the secant and cosecant functions and can graph them. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 7.0 Students know that the tangent of the angle that a line makes with the x-axis is equal to the slope of the line. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| 8.0 Students know the definitions of the inverse trigonometric functions and can graph the functions. | 5.1.d Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |
| 9.0 Students compute, by hand, the values of the trigonometric functions and the inverse trigonometric functions at various standard points. | 5.1.c Analyze properties of trigonometric functions in a variety of ways (e.g., graphing and solving problems) | Math 1300, 1304, 1305, 2304, 3331 Appendix 0, 1, 11 - Domain 5 |
| | 5.1.d Know and apply the definitions and properties of inverse trigonometric functions (i.e., arcsin, arccos, and arctan) | Math 1300, 1304, 1305, 2304 Appendix 0, 1, 11 - Domain 5 |
| 10.0 Students demonstrate an understanding of the addition formulas for sines and cosines and their proofs and can use those formulas to prove and/or simplify other trigonometric identities. | 5.1.b Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle) | Math 1300, 1304 Appendix 0 and 11- Domain 5 |
| 11.0 Students demonstrate an understanding of half-angle and double-angle formulas for sines and cosines and can use those formulas to prove and/or simplify other trigonometric identities. | 5.1.b Prove the sine, cosine, and tangent sum formulas for all real values, and derive special applications of the sum formulas (e.g., double angle, half angle) | Math 1300, 1304 Appendix 0 and 11- Domain 5 |
| 12.0 Students use trigonometry to determine unknown sides or angles in right triangles. | 2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |

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| 13.0 Students know the law of sines and the law of cosines and apply those laws to solve problems. | 2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 14.0 Students determine the area of a triangle, given one angle and the two adjacent sides. | 2.2.b Understand, apply, and justify properties of triangles (e.g., the Exterior Angle Theorem, concurrence theorems, trigonometric ratios, Triangle Inequality, Law of Sines, Law of Cosines, the Pythagorean Theorem and its converse) | Math 1300, 1304, 1305, 2101, 3215, 3300 Appendix 0 and 11 - Domain 2 |
| 15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 15.0 Students are familiar with polar coordinates. In particular, they can determine polar coordinates of a point given in rectangular coordinates and vice versa. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 16.0 Students represent equations given in rectangular coordinates in terms of polar coordinates. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 17.0 Students are familiar with complex numbers. They can represent a complex number in polar form and know how to multiply complex numbers in their polar form. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 18.0 Students know DeMoivre’s theorem and can give nth roots of a complex number given in polar form. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 19.0 Students are adept at using trigonometry in a variety of applications and word problems. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 1300, 1304, 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| **MATHEMATICAL ANALYSIS** This discipline combines many of the trigonometric, geometric, and algebraic techniques needed to prepare students for the study of calculus and strengthens their conceptual understanding of problems and mathematical reasoning in solving problems. These standards take a functional point of view toward those topics. The most significant new concept is that of limits. Mathematical analysis is often combined with a course in trigonometry or perhaps with one in linear algebra to make a year-long precalculus course. | | |

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| 1.0 Students are familiar with, and can apply, polar coordinates and vectors in the plane. In particular, they can translate between polar and rectangular coordinates and can interpret polar coordinates and vectors graphically. | 1.4.a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 2.0 Students are adept at the arithmetic of complex numbers. They can use the trigonometric form of complex numbers and understand that a function of a complex variable can be viewed as a function of two real variables. They know the proof of DeMoivre's theorem. | 5.1.e Understand and apply polar representations of complex numbers (e.g., DeMoivre's Theorem) | Math 3000, 3121, 4901 Appendix 0, 1 and 11 - Domain 5 |
| 3.0 Students can give proofs of various formulas by using the technique of mathematical induction. | 1.2.b Prove and use the following:<br>• The Rational Root Theorem for polynomials with integer coefficients<br>• The Factor Theorem<br>• The Conjugate Roots Theorem for polynomial equations with real coefficients<br>• The Quadratic Formula for real and complex quadratic polynomials<br>• The Binomial Theorem | Math 1130, 3000, 3121, 4901 Appendix 0 and 11 - Domain 1 |
| | 3.1.b Use the Principle of Mathematical Induction to prove results in number theory | Math 3000, 3121, 3600 Appendix 0 and 11 -Domain 3 |
| 4.0 Students know the statement of, and can apply, the fundamental theorem of algebra. | 1.2 Polynomial Equations and Inequalities<br>c. Analyze and solve polynomial equations with real coefficients using the Fundamental Theorem of Algebra | Math 1130, 1304, 2101 Appendix 0 and 11 - Domain 1 |
| 5.0 Students are familiar with conic sections, both analytically and geometrically: | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |
| | 2.3.b Understand, apply, and justify properties of three-dimensional objects from an advanced standpoint (e.g., derive the volume and surface area formulas for prisms, pyramids, cones, cylinders, and spheres) | Math 1305, 2304 Appendix 0 and 11 - Domain 2 |

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| 5.1 Students can take a quadratic equation in two variables; put it in standard form by completing the square and using rotations and translations, if necessary; determine what type of conic section the equation represents; and determine its geometric components (foci, asymptotes, and so forth). | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |
| 5.2 Students can take a geometric description of a conic section—for example, the locus of points whose sum of its distances from (1, 0) and (-1, 0) is 6—and derive a quadratic equation representing it. | 2.2.e Use techniques in coordinate geometry to prove geometric theorems | Math 1300, 3215 Appendix 0 and 11 |
| 6.0 Students find the roots and poles of a rational function and can graph the function and locate its asymptotes. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| 7.0 Students demonstrate an understanding of functions and equations defined parametrically and can graph them. | 1.3.b Analyze properties of polynomial, rational, radical, and absolute value functions in a variety of ways (e.g., graphing, solving problems) | Math 1130, 1304, 1305, 2304, 3300 Appendix 0 and 11-Domain 1 |
| Students are familiar with the notion of the limit of a sequence and the limit of a function as the independent variable approaches a number or infinity. They determine whether certain sequences converge or diverge. | 5.2.a Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | Math 1304, 1305, 2304, 3000, 3300 Appendix 0 and 11 - Domain 5 |
| **LINEAR ALGEBRA** The general goal in this discipline is for students to learn the techniques of matrix manipulation so that they can solve systems of linear equations in any number of variables. Linear algebra is most often combined with another subject, such as trigonometry, mathematical analysis, or precalculus. | | |
| 1.0 Students solve linear equations in any number of variables by using Gauss-Jordan elimination. | 1.4.c Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | Math 2101 Appendix 0 and 11 - Domain 1 |
| 2.0 Students interpret linear systems as coefficient matrices and the Gauss-Jordan method as row operations on the coefficient matrix. | 1.4.c Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | Math 2101 Appendix 0 and 11 - Domain 1 |

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| 3.0 Students reduce rectangular matrices to row echelon form. | 1.4.a. Understand and apply the geometric interpretation and basic operations of vectors in two and three dimensions, including their scalar multiples and scalar (dot) and cross products | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| 4.0 Students perform addition on matrices and vectors. | 1.4.b Prove the basic properties of vectors (e.g., perpendicular vectors have zero dot product) | Math 2304, 2101 Appendix 0 and 11 - Domain 1 |
| 5.0 Students perform matrix multiplication and multiply vectors by matrices and by scalars. | 1.4.c Understand and apply the basic properties and operations of matrices and determinants (e.g., to determine the solvability of linear systems of equations) | Math 2101 Appendix 0 and 11 - Domain 1 |
| **PROBABILITY AND STATISTICS** This discipline is an introduction to the study of probability, interpretation of data, and fundamental statistical problem solving. Mastery of this academic content will provide students with a solid foundation in probability and facility in processing statistical information. | | |
| 1.0 Students know the definition of the notion of independent events and can use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces. | 4.1.a Prove and apply basic principles of permutations and combinations | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| | 4.1.b Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles) | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 2.0 Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces. | 4.1.c Use and explain the concept of conditional probability | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 3.0 Students demonstrate an understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes, such as the probability of the occurrence of five heads in 14 coin tosses. | 4.1.d Interpret the probability of an outcome | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 4.0 Students are familiar with the standard distributions (normal, binomial, and exponential) and can use them to solve for events in problems in which the distribution belongs to those families. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |

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| 5.0 Students determine the mean and the standard deviation of a normally distributed random variable. | 4.2.a Compute and interpret the mean, median, and mode of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 6.0 Students know the definitions of the *mean*, *median*, and *mode* of a distribution of data and can compute each in particular situations. | | |
| 7.0 Students compute the variance and the standard deviation of a distribution of data. | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 8.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots. | 4.2.c Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| **ADVANCED PLACEMENT PROBABILITY AND STATISTICS** This discipline is a technical and in-depth extension of probability and statistics. In particular, mastery of academic content for advanced placement gives students the background to succeed in the Advanced Placement examination in the subject. | | |
| 1.0 Students solve probability problems with finite sample spaces by using the rules for addition, multiplication, and complementation for probability distributions and understand the simplifications that arise with independent events. | 4.1.a Prove and apply basic principles of permutations and combinations | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| | 4.1.b Illustrate finite probability using a variety of examples and models (e.g., the fundamental counting principles) | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 2.0 Students know the definition of *conditional probability* and use it to solve for probabilities in finite sample spaces. | 4.1.c Use and explain the concept of conditional probability | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 3.0 Students demonstrate an understanding of the notion of discrete random variables by using this concept to solve for the probabilities of outcomes, such as the probability of the occurrence of five or fewer heads in 14 coin tosses. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 4.0 Students understand the notion of a continuous random variable and can interpret the probability of an outcome as the area of a region under the graph of the probability density function associated with the random variable. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 5.0 Students know the definition of the *mean of a discrete random variable* and can determine the mean for a particular discrete random variable. | 4.2.a Compute and interpret the mean, median, and mode of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |

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| 6.0 Students know the definition of the *variance of a discrete random variable* and can determine the variance for a particular discrete random variable. | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 7.0 Students demonstrate an understanding of the standard distributions (normal, binomial, and exponential) and can use the distributions to solve for events in problems in which the distribution belongs to those families. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 8.0 Students determine the mean and the standard deviation of a normally distributed random variable. | 4.2.a Compute and interpret the mean, median, and mode of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 9.0 Students know the central limit theorem and can use it to obtain approximations for probabilities in problems of finite sample spaces in which the probabilities are distributed binomially. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| 10.0 Students know the definitions of the *mean, median, and mode of distribution* of data and can compute each of them in particular situations. | 4.2.a Compute and interpret the mean, median, and mode of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 11.0 Students compute the variance and the standard deviation of a distribution of data. | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 12.0 Students find the line of best fit to a given distribution of data by using least squares regression. | 4.2.d Know the method of least squares and apply it to linear regression and correlation | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 13.0 Students know what the *correlation coefficient of two variables* means and are familiar with the coefficient's properties. | 4.2.d Know the method of least squares and apply it to linear regression and correlation | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 14.0 Students organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line graphs and bar graphs, stem-and-leaf displays, scatterplots, and box-and-whisker plots. | 4.2.c Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 15.0 Students are familiar with the notions of a statistic of a distribution of values, of the sampling distribution of a statistic, and of the variability of a | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |

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| statistic. | 4.2.c Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 16.0 Students know basic facts concerning the relation between the mean and the standard deviation of a sampling distribution and the mean and the standard deviation of the population distribution. | 4.2.a Compute and interpret the mean, median, and mode of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| | 4.2.b Compute and interpret quartiles, range, variance, and standard deviation of both discrete and continuous distributions | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| | 4.2.c Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 17.0 Students determine confidence intervals for a simple random sample from a normal distribution of data and determine the sample size required for a desired margin of error. | 4.2.c Select and evaluate sampling methods appropriate to a task (e.g., random, systematic, cluster, convenience sampling) and display the results<br>4.2.e Know and apply the chi-square test | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 18.0 Students determine the P-value for a statistic for a simple random sample from a normal distribution. | | Statistics 3502 Appendix 0 and 11 - Domain 4 |
| 19.0 Students are familiar with the *chi*-square distribution and *chi*-square test and understand their uses. | 4.1.e Use normal, binomial, and exponential distributions to solve and interpret probability problems | Statistics 3401 Appendix 0 and 11 - Domain 4 |
| **CALCULUS** When taught in high school, calculus should be presented with the same level of depth and rigor as are entry-level college and university calculus courses. These standards outline a complete college curriculum in one variable calculus. Many high school programs may have insufficient time to cover all of the following content in a typical academic year. For example, some districts may treat differential equations lightly and spend substantial time on infinite sequences and series. Others may do the opposite. Consideration of the College Board syllabi for the Calculus AB and Calculus BC sections of the Advanced Placement Examination in Mathematics may be helpful in making curricular decisions. Calculus is a widely applied area of mathematics and involves a beautiful intrinsic theory. Students mastering this content will be exposed to both aspects of the subject. | | |
| 1.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity: | 5.2.a Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | Math 1304, 1305, 2304, 3000, 3300 Appendix 0 and 11 - Domain 5 |

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| 1.1 Students prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions. | 5.2.a Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | Math 1304, 1305, 2304, 3000, 3300 Appendix 0 and 11 - Domain 5 |
| 1.2 Students use graphical calculators to verify and estimate limits. | 5.2.a Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | Math 1304, 1305, 2304, 3000, 3300 Appendix 0 and 11 - Domain 5 |
| 1.3 Students prove and use special limits, such as the limits of (sin(x))/x and (1-cos(x))/x as x tends to 0. | 5.2.a Derive basic properties of limits and continuity, including the Sum, Difference, Product, Constant Multiple, and Quotient Rules, using the formal definition of a limit | Math 1304, 1305, 2304, 3000, 3300 Appendix 0 and 11 - Domain 5 |
| 2.0 Students demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function. | 5.2.c Know and apply the Intermediate Value Theorem, using the geometric implications of continuity | Math 1304, 3300 Appendix 0 and 11 - Domain 5 |
| 3.0 Students demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem. | 5.2.c Know and apply the Intermediate Value Theorem, using the geometric implications of continuity | Math 1304, 3300 Appendix 0 and 11 - Domain 5 |
| 4.0 Students demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability: | 5.3.b Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 4.1 Students demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function. | 5.3.b Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 4.2 Students demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change. Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function. | 5.3.b Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| | 5.3.d Use the derivative to solve rectilinear motion, related rate, and optimization problems | Math 1304 Appendix 0 and 11 - Domain 5 |
| 4.3 Students understand the relation between differentiability and continuity. | 5.3.c Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule | Math 1304, 3300 Appendix 0 and 11 - Domain 5 |

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| 4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions. | 5.3.a Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative | Math 1304 Appendix 0 and 11 - Domain 5 |
| 5.0 Students know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions. | 5.3.b Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 6.0 Students find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth. | 5.3.d Use the derivative to solve rectilinear motion, related rate, and optimization problems | Math 1304 Appendix 0 and 11 - Domain 5 |
| 7.0 Students compute derivatives of higher orders. | 5.3.a Derive the rules of differentiation for polynomial, trigonometric, and logarithmic functions using the formal definition of derivative 5.3(e) | Math 1304 Appendix 0 and 11 - Domain 5 |
| 8.0 Students know and can apply Rolle's theorem, the mean value theorem, and L'Hôpital's rule. | 5.3.c Interpret both continuous and differentiable functions geometrically and analytically and apply Rolle's Theorem, the Mean Value Theorem, and L'Hopital's rule | Math 1304, 3300 Appendix 0 and 11 - Domain 5 |
| 9.0 Students use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing. | 5.3.e Use the derivative to analyze functions and planar curves (e.g., maxima, minima, inflection points, concavity) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 10.0 Students know Newton's method for approximating the zeros of a function. | 5.3.b Interpret the concept of derivative geometrically, numerically, and analytically (i.e., slope of the tangent, limit of difference quotients, extrema, Newton's method, and instantaneous rate of change) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 11.0 Students use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts. | 5.3.d Use the derivative to solve rectilinear motion, related rate, and optimization problems | Math 1304 Appendix 0 and 11 - Domain 5 |
| 12.0 Students use differentiation to solve related rate problems in a variety of pure and applied contexts. | 5.3.d Use the derivative to solve rectilinear motion, related rate, and optimization problems | Math 1304 Appendix 0 and 11 - Domain 5 |
| 13.0 Students know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals. | 5.4.b Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |

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| 14.0 Students apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 15.0 Students demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives. | 5.4.c Prove the Fundamental Theorem of Calculus, and use it to interpret definite integrals as antiderivatives | Math 1305 Appendix 0 and 11 - Domain 5 |
| 16.0 Students use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 17.0 Students compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 18.0 Students know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 19.0 Students compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 20.0 Students compute the integrals of trigonometric functions by using the techniques noted above. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically. | 5.4.b Interpret the concept of a definite integral geometrically, numerically, and analytically (e.g., limit of Riemann sums) | Math 1304, 2304 Appendix 0 and 11 - Domain 5 |
| 22.0 Students understand improper integrals as limits of definite integrals. | 5.4.d Apply the concept of integrals to compute the length of curves and the areas and volumes of geometric figures | Math 1305 Appendix 0 and 11 - Domain 5 |
| 23.0 Students demonstrate an understanding of the definitions of convergence and divergence of sequences and series of real numbers. By using such tests as the comparison test, ratio test, and alternate series test, they can determine whether a series converges. | 5.5.a Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) | Math 1305, 2304, 3300, Appendix 0 and 11 - Domain 5 |

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| 24.0 Students understand and can compute the radius (interval) of the convergence of power series. | 5.5.b Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests) | Math 2304, 3300, Appendix 0 and 11 - Domain 5 |
| 25.0 Students differentiate and integrate the terms of a power series in order to form new series from known ones. | 5.5.a Derive and apply the formulas for the sums of finite arithmetic series and finite and infinite geometric series (e.g., express repeating decimals as a rational number) | Math 1305, 2304, 3300, Appendix 0 and 11 - Domain 5 |
| | 5.5.b Determine convergence of a given sequence or series using standard techniques (e.g., Ratio, Comparison, Integral Tests) | Math 2304, 3300, Appendix 0 and 11 - Domain 5 |
| 26.0 Students calculate Taylor polynomials and Taylor series of basic functions, including the remainder term. | 5.5.c Calculate Taylor series and Taylor polynomials of basic functions | Math 2304 Appendix 0 and 11 - Domain 5 |
| 27.0 Students know the techniques of solution of selected elementary differential equations and their applications to a wide variety of situations, including growth-and-decay problems. | 5.3.f Solve separable first-order differential equations and apply them to growth and decay problems | Math 1305, 3331 Appendix 0, 11 - Domain 5, 12 |